Photonic crystals: from Bloch modes to T-matrices

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Objectives :

- Links between Bloch modes and grating theories (transfer matrix)
- Study of anomalous refraction at the band edges and applications

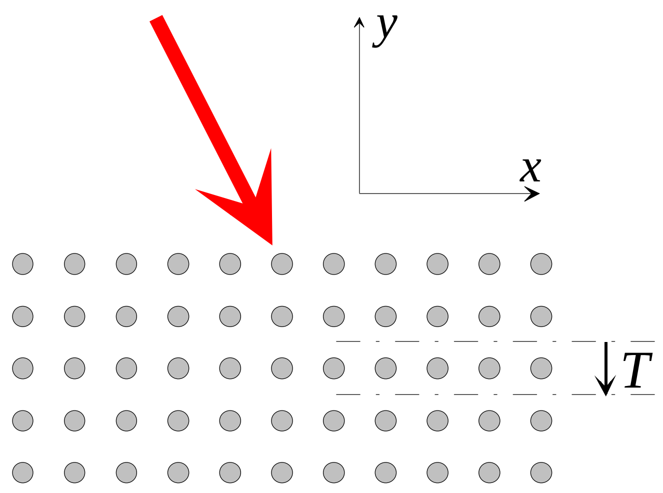
Bloch wave: $\psi_{\vec{k}}(\vec{r}) = e^{i\,\vec{k}\cdot\vec{r}}\;u_{\vec{k}}(\vec{r})$

➔ Dispersion relation of the Bloch modes

But:

- the crystal is infinite and fills the whole space
- no incident field
- only real Bloch vectors are considered

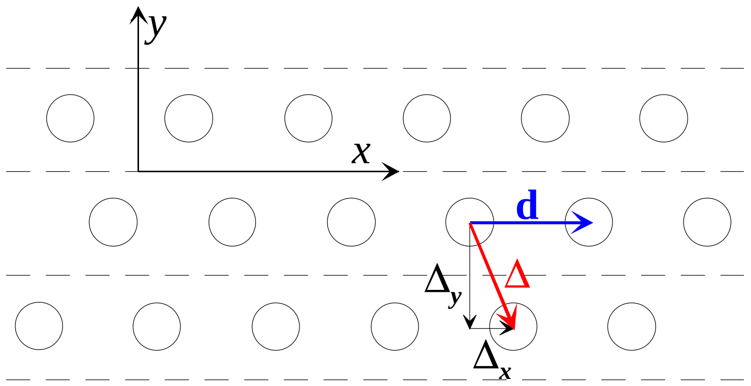
Studied structure:



2D photonic crystal made of dielectric rods of radius 0.475, with optical index n = 3 arranged on a square lattice with period d = 1.27

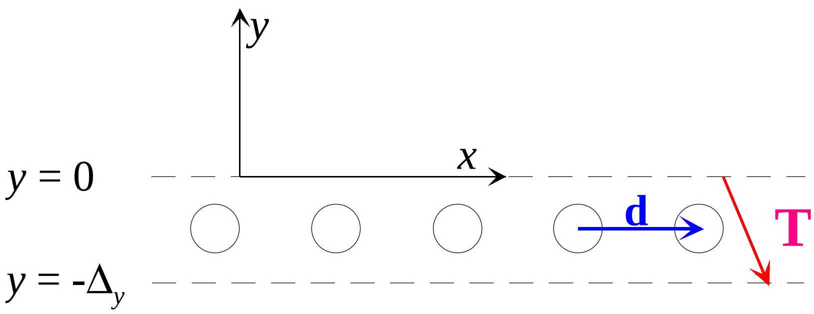
The eigenvalues of the transfer matrix T give us more information than the Bloch modes.

T-matrix and Bloch modes



Bloch mode: solution with only phase shifts for the elementary translations **d** and Δ.

A crystal made of 3 gratings. Infinite along the x-direction. Finite extent with respect to the y-direction. Plenty of place for an incident field !



T-matrix point of view:
d translation gives phase shift (pseudo-periodicity).
Δ translation gives phase shift for eigenvectors of the **T** operator if the eigenvalue has a modulus equal to 1.

Harmonic problem → ω given.
Pseudo-periodic component of the field → k_x given:

$$u(x+d,y)=\exp(\mathrm{i}\,k_x\,d)\,u(x,y).$$

Eigenvector of the T-matrix, with eigenvalue μ :

$$\mathrm{T}\,u=\mu\,u$$

$$\mathrm{T}\,u=\exp(\mathrm{i}\,\arg(\mu))\,u \quad , \quad \text{if } |\mu| = 1$$

Each eigenvalue μ of T with |μ| = 1 is associated with a propagating Bloch mode. The *y*-component of the Bloch vector is:

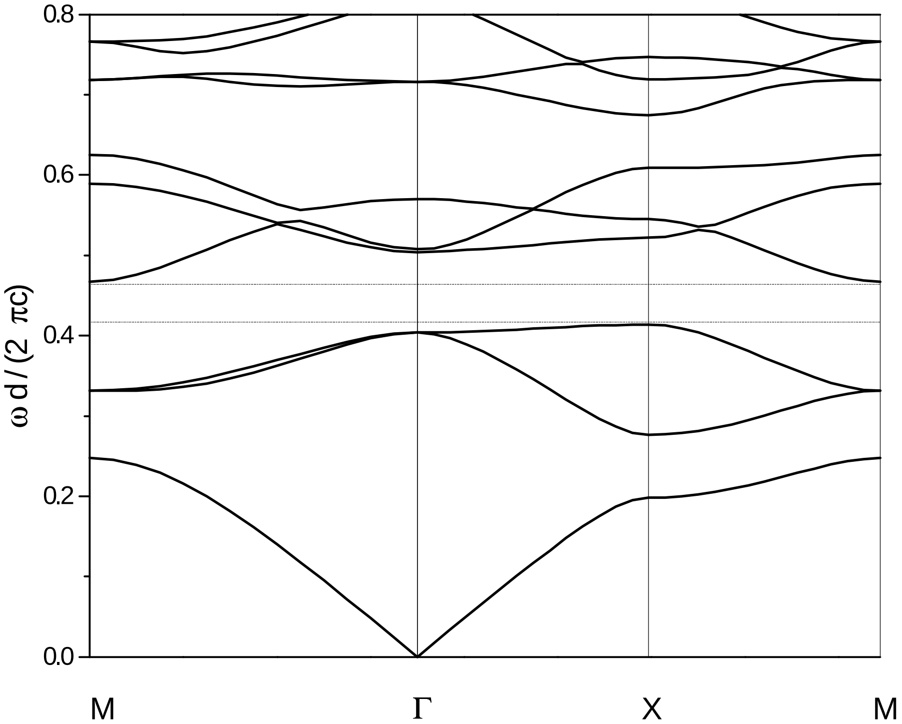
$$k_y=\frac{k_x\Delta_x-\arg(\mu)}{\Delta_y}$$

$$\left.\begin{array}{r}\omega \text{ given}\\ k_x \text{ given}\end{array}\right\}\xrightarrow{\text{grating problem}} k_y$$

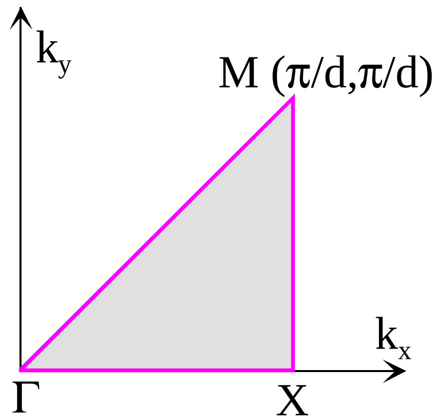
Dispersion curve of Bloch modes

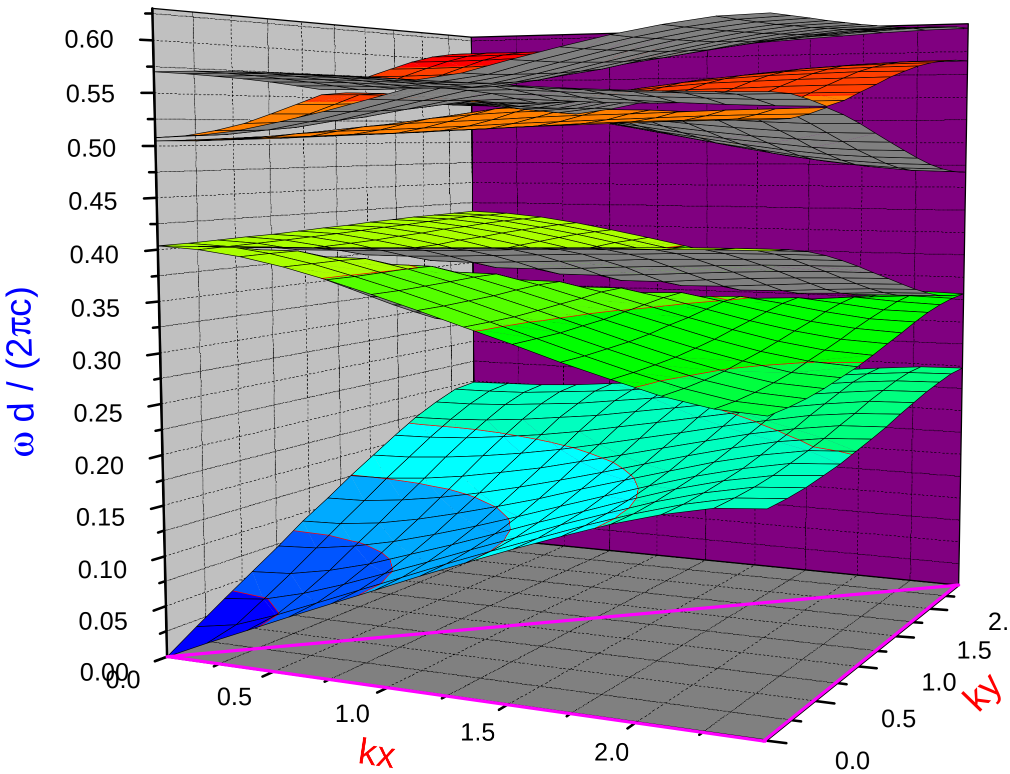
Dispersion relation of the Bloch modes in a 2D photonic crystal

In the infinite structure the field is can be represented as a sum of Bloch modes: $\psi_{\vec{k}}(\vec{r}) = e^{i\,\vec{k}\cdot\vec{r}}\; u_{\vec{k}}(\vec{r})$

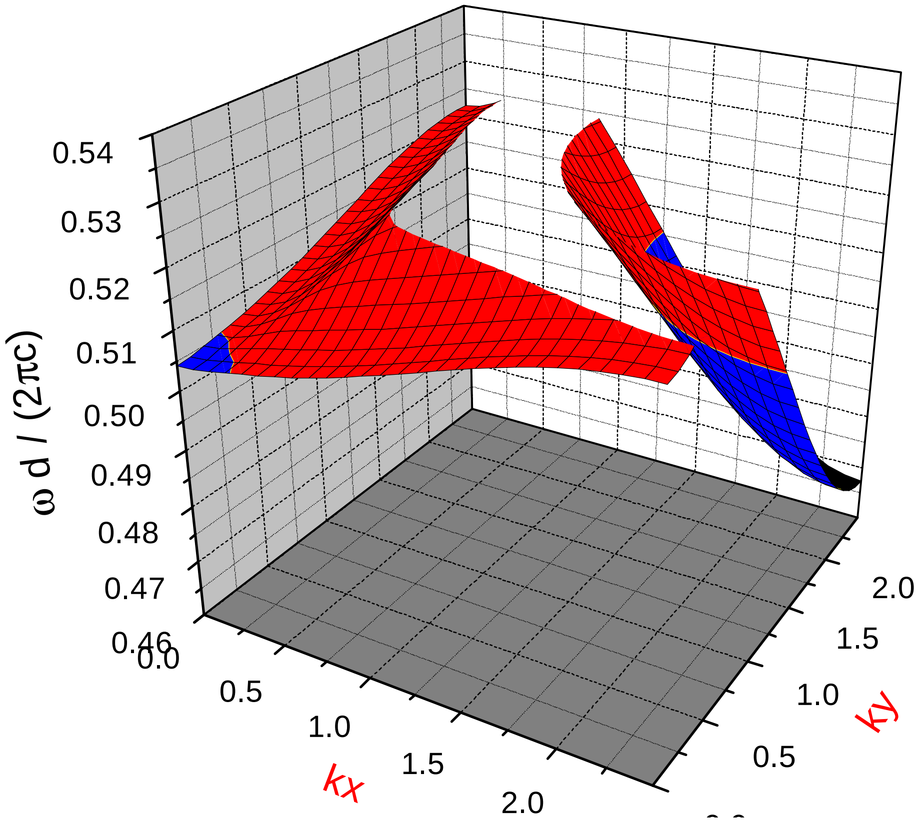


Dispersion diagram for E// polarization. The first reduced Brillouin zone is represented below.



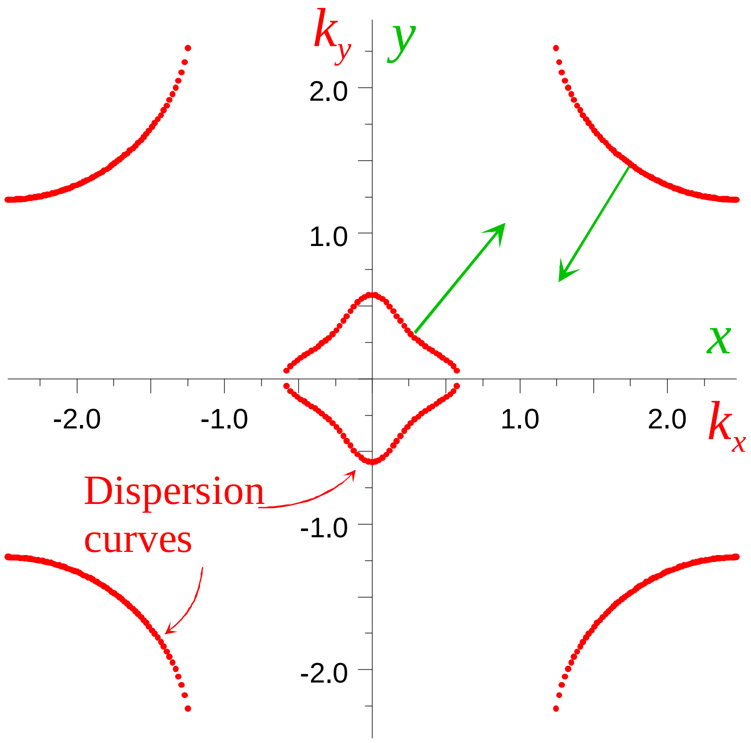


3D view of the dispersion diagram of the Bloch modes.



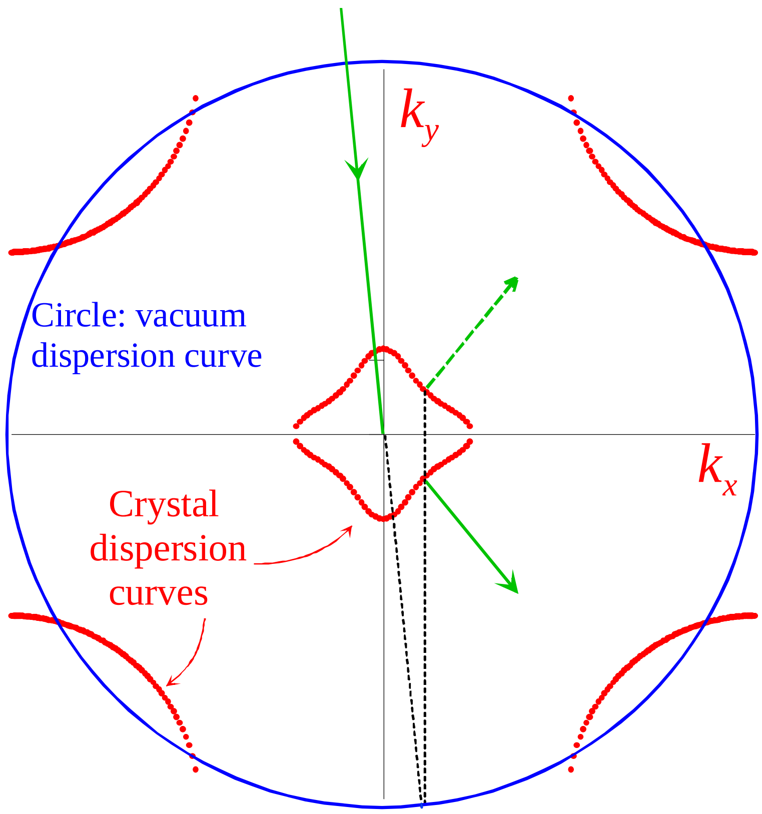
Enlargement of the previous figure at the upper limit of the second band gap.

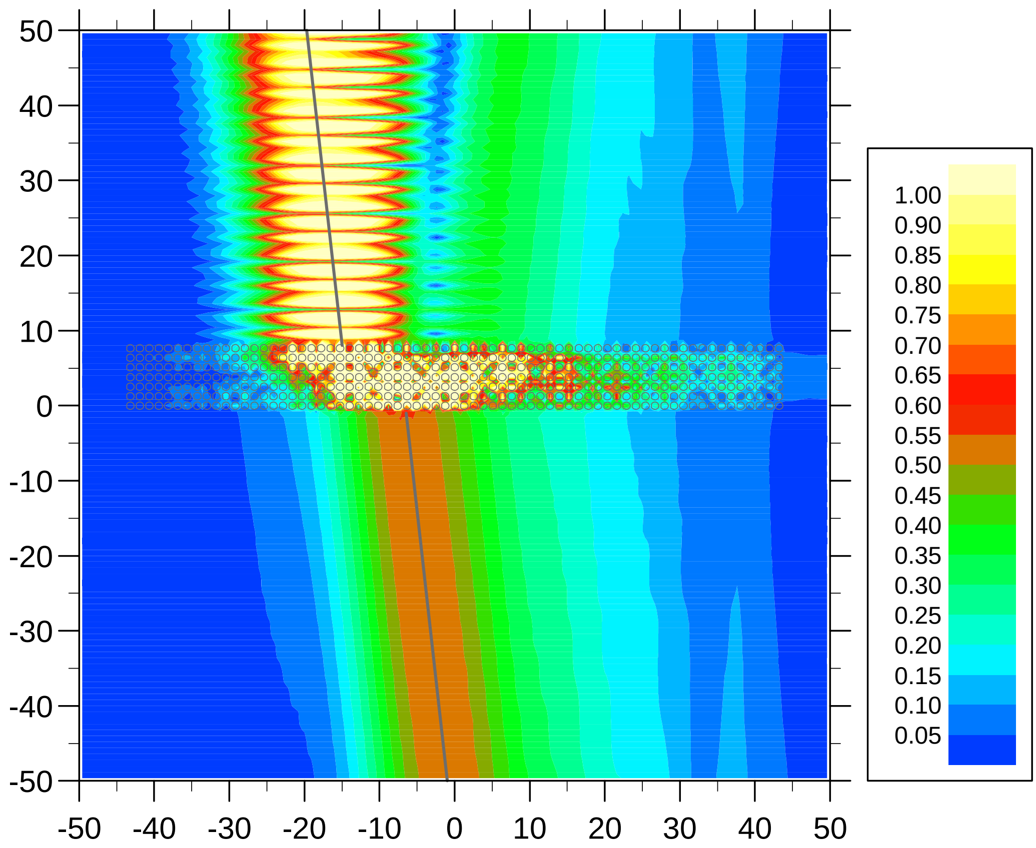
For a fixed wavelength λ the dispersion relation is the intersection between a horizontal plane and the sheets.



The energy flow is given by $\dfrac{\partial \omega}{\partial k_x}\vec{e}_x + \dfrac{\partial \omega}{\partial k_y}\vec{e}_y$: it is perpendicular to the curve and points toward the ascending side of the sheet.

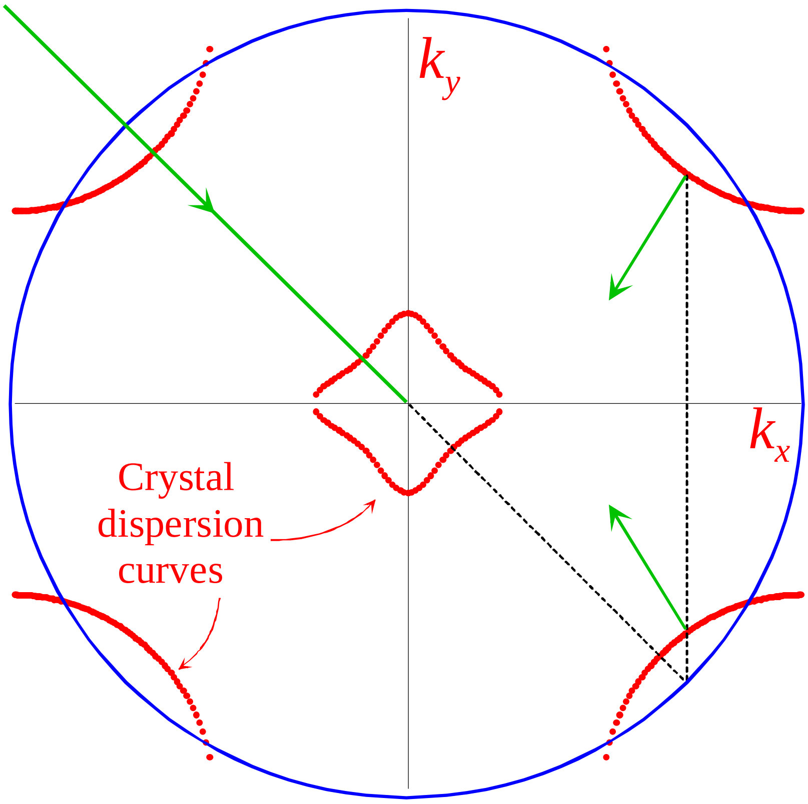
Ultra-refraction

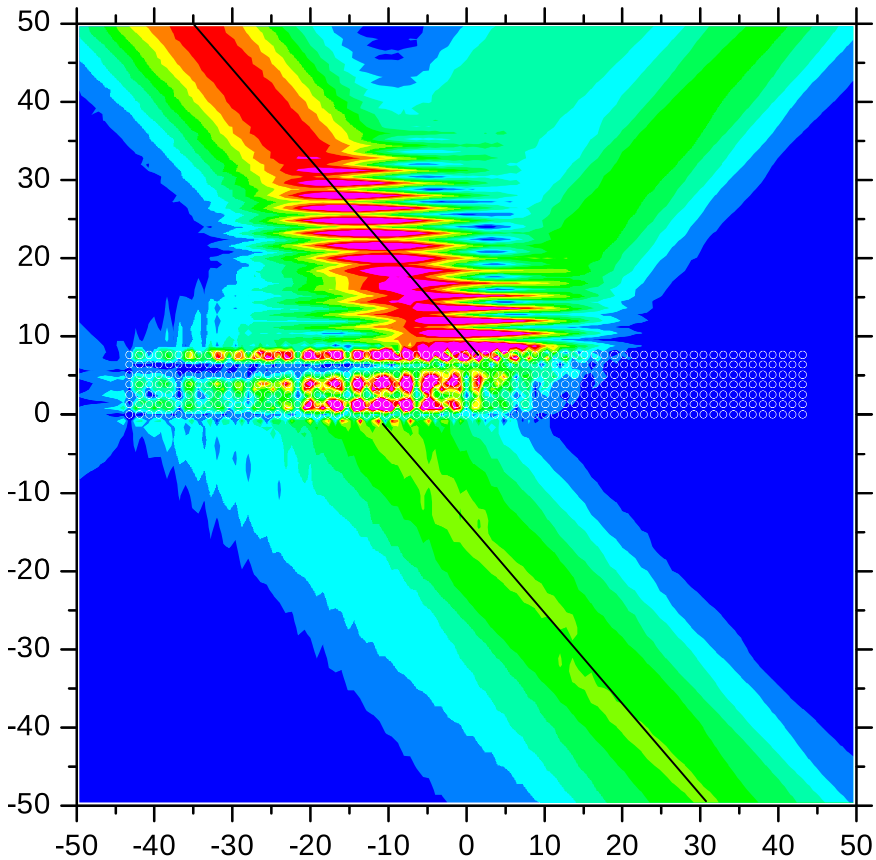




Bounded 2D photonic crystal
Modulus of the electric field. $\lambda = 2.54$, $\theta = 6.4°$

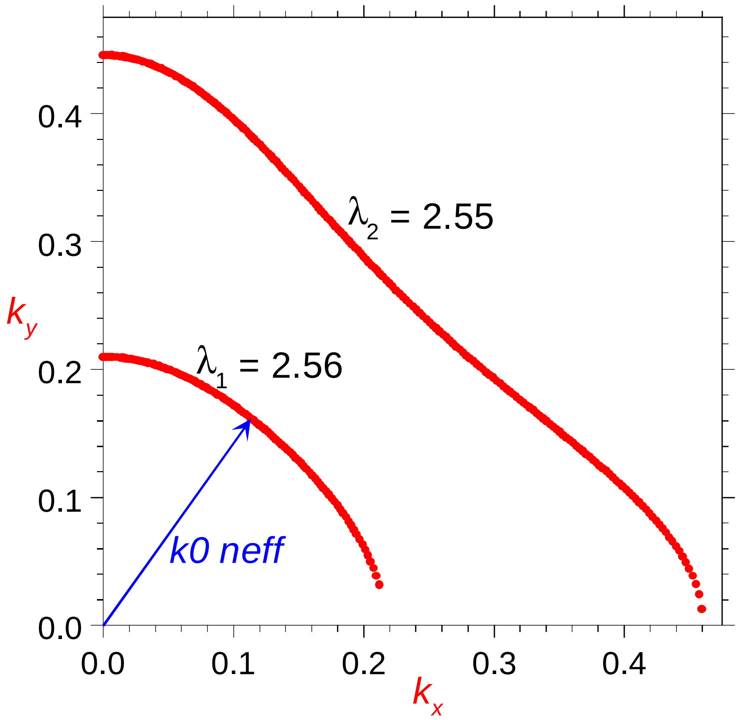
Negative refraction



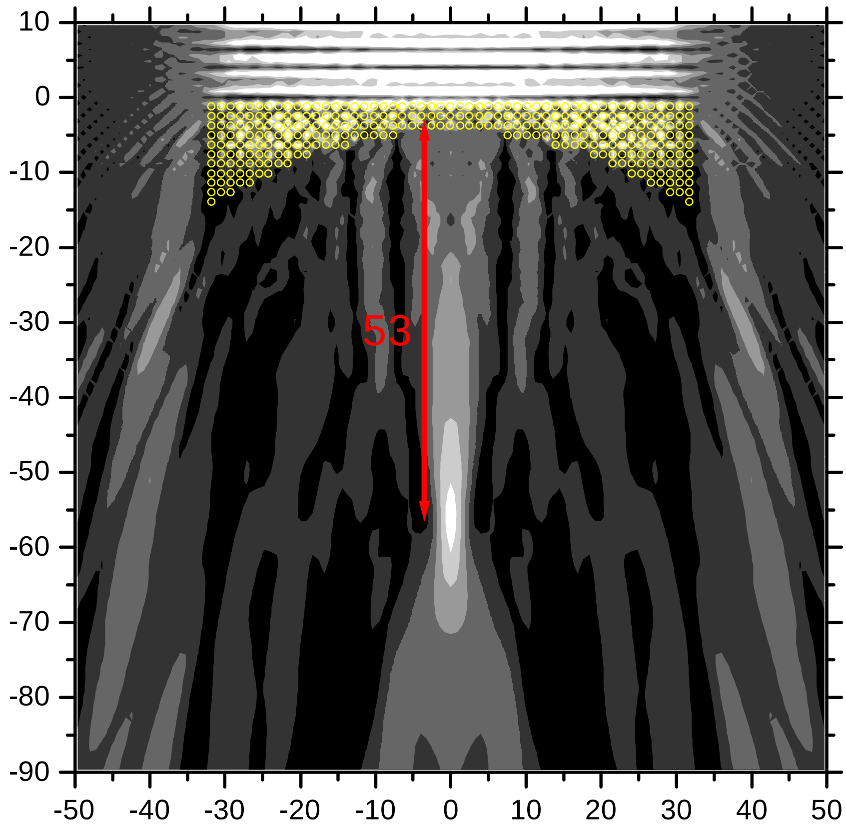


Modulus of the electric field. $\lambda = 2.54$, $\theta = 40°$

Application: ultrarefractive optical components
1 - lens



With $\lambda = 2.56$, the curve is almost a circle and simulates a homogeneous material with optical index $n_{eff} = 0.086$



R: radius of curvature of the concave side.
Focal length:

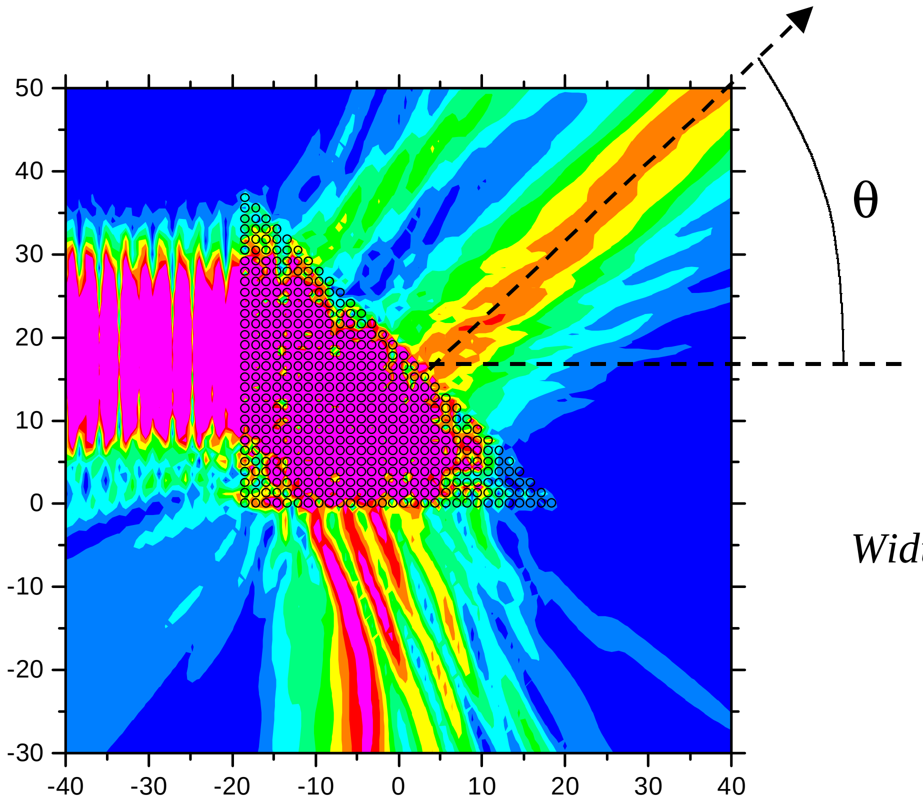
$$f = \frac{R}{1 - n_{eff}}$$

$$= \frac{50}{1 - 0.086} = 55$$

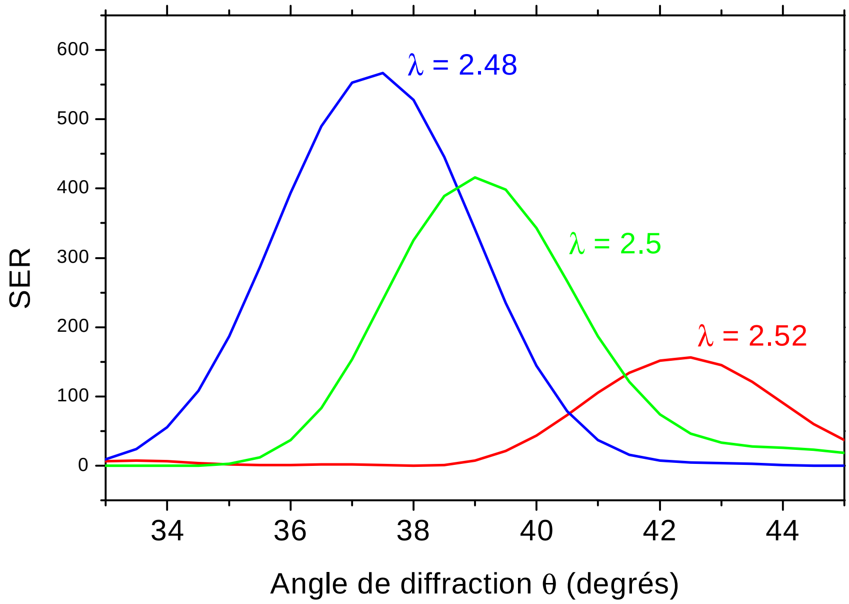
"Ultrarefractive" microlens: a convergent lens with focal length = 21 λ. The width of the lens is about 25λ.

→ Very high index contrast $(1/0.086 \approx 12)$

Application: ultrarefractive optical components
2 - microprism



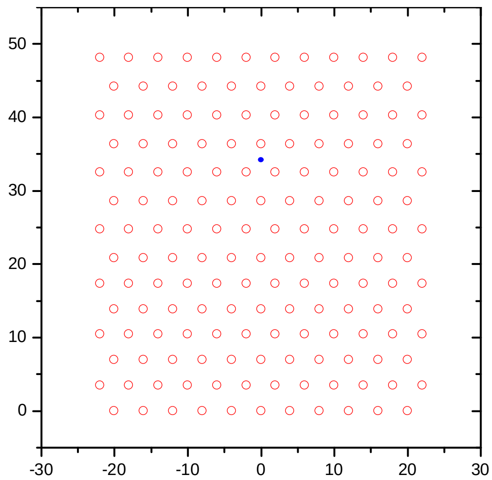
Width of the prism = 14 λ



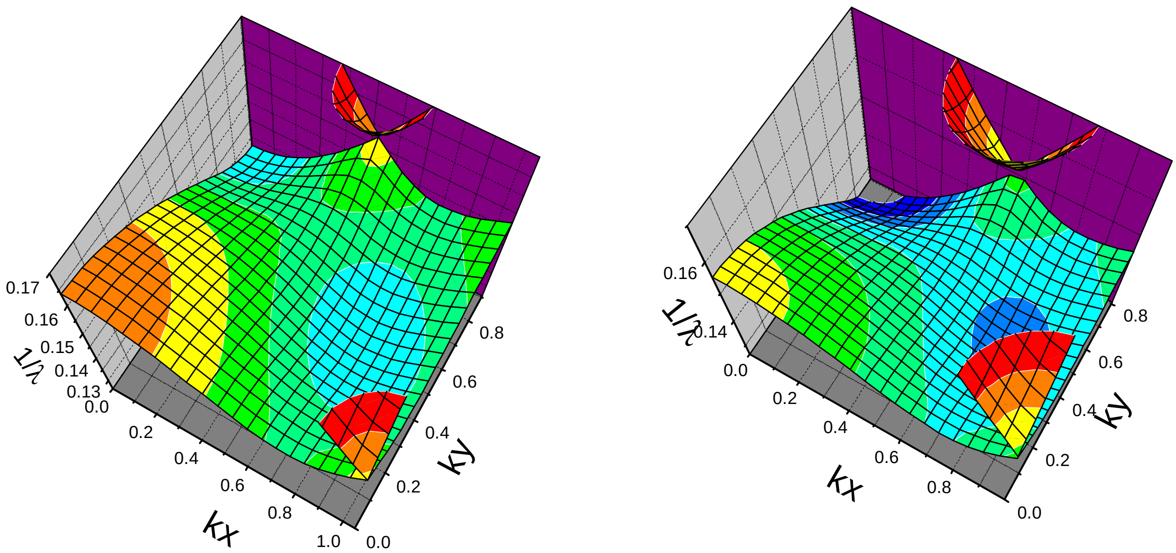
Band edge ➔ rapid variations with the wavelength.
The photonic crystal prism is more dispersive than usual prisms or gratings.

→ Very dispersive material.

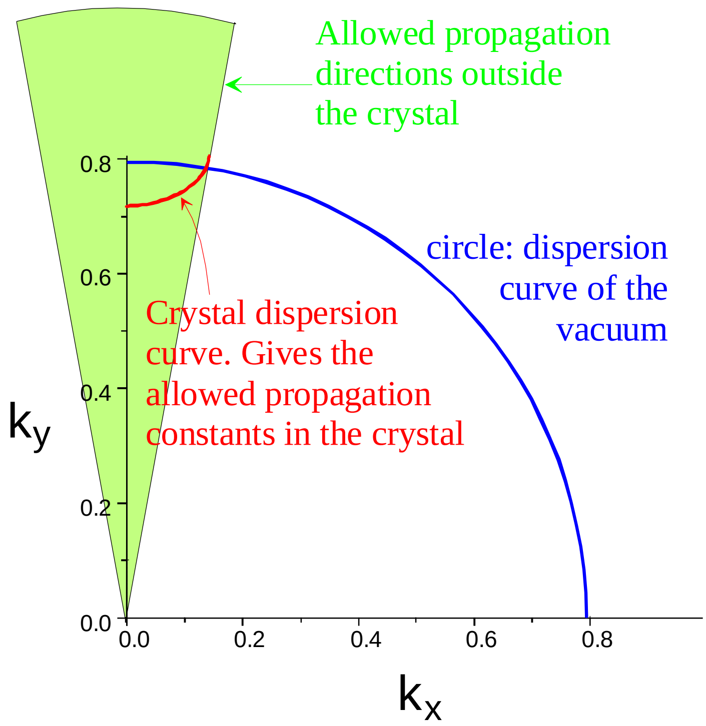
3 - Directive sources

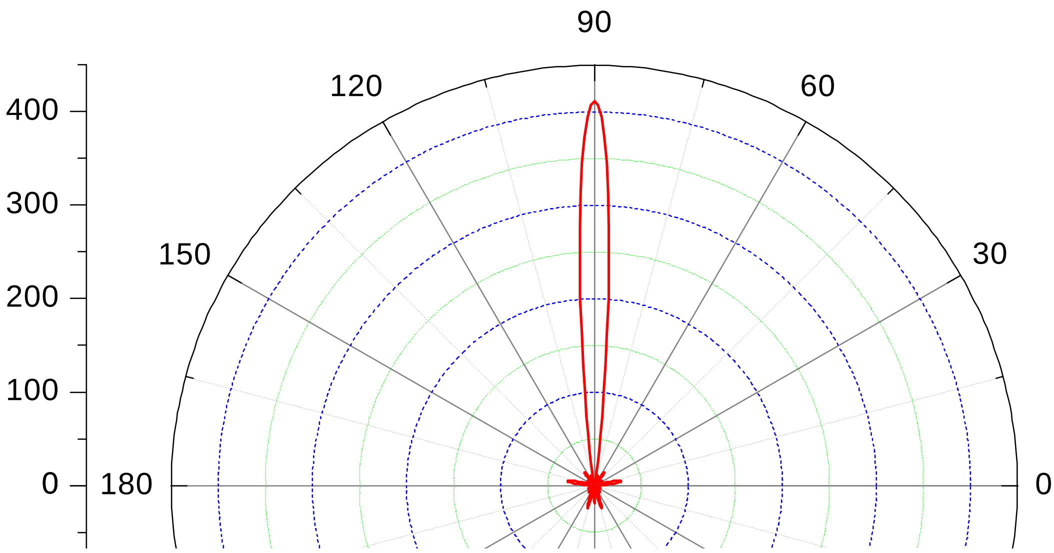


The hexagonal lattice has been expanded along the y direction (× 1.126) in the upper region of the crystal. 161 dielectric rods with radius r = 0.6, optical index n = 2.9. A wire source is located at x = 0, y = 34.

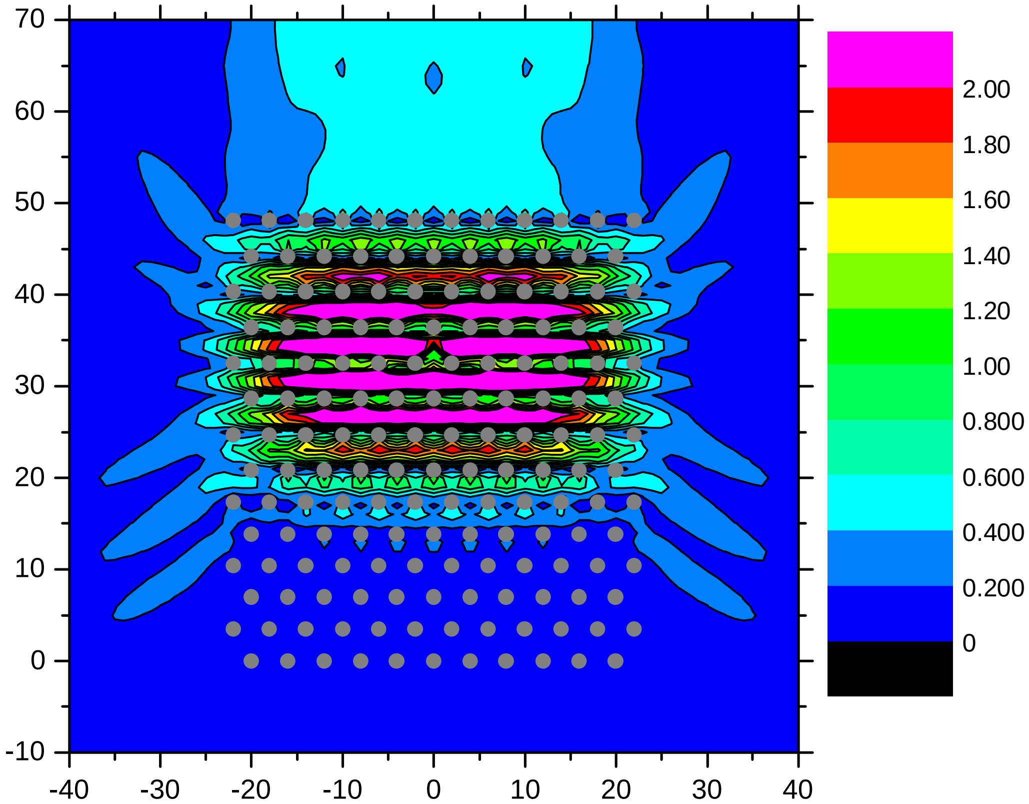


Dispersion relation of the Bloch modes for the hexagonal (left) and the expanded lattice (right). The bottom plane of the figure is for λ = 7.93.





Radiation pattern of the photonic crystal source λ = 7.93.



Modulus of the electric field. Wire source is located at x = 0, y =34. λ = 7.93.

The computation time (161 rods) is about 5 seconds to compute the coefficients of the field and 20 seconds to compute the field map (80x80 points) on a desktop computer.

Conclusion

<table>
<tr><th>Classical Bloch study</th><th>Grating, T-matrix</th><th>More realistic problem</th></tr>
<tr><td>Infinite crystal filling the entire space</td><td>Slice of grating.
Finite in one direction</td><td>Finite structure (microlens, microprism, antenna,…)</td></tr>
<tr><td>No incident field</td><td>Simple link with the incident field (diffraction case, emitting situation)</td><td>Incident field (plane wave, limited beam, source in the crystal)</td></tr>
<tr><td>Only propagating modes</td><td>Also evanescent modes.
→ information on the transmission inside a bandgap</td><td>Also evanescent modes</td></tr>
<tr><td>Dispersion relations of the Bloch modes
$k_x, k_y \rightarrow \omega$</td><td>Eigenvalues of the T-matrix
$\omega, k_x \rightarrow k_y$</td><td></td></tr>
<tr><td></td><td>Includes all the features of a classical Bloch study, plus more…
More accurate, faster.</td><td></td></tr>
<tr><td colspan="2">Synthetic approach (visual support given by the 3D dispersion diagrams of the Bloch waves). Helpful in the understanding of the photonic crystal properties.
Permits to obtain easily the parameters giving rise to:
• ultrarefraction (the photonic crystal simulates a homogeneous material with a very low optical index),
• negative refraction,
and more generally to get the allowed directions for the mean energy flux in the photonic crystal.</td><td>Rigorous numerical computations
→ Confirm the expected behaviors
→ Quantitative results
→ Check the effects of the boundaries of the photonic crystal</td></tr>
</table>

References

B. Gralak, S. Enoch and G. Tayeb. "Anomalous refractive properties of photonic crystals." J. Opt. Soc. Am. A **17**, 1012-1020 (2000).

S. Enoch, G. Tayeb and D. Maystre, "Numerical evidence of ultrarefractive optics in photonic crystals", *Optics Communications* **161**, 171-176 (1999)

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