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# Experimental and theoretical study of resonant microcavities in two-dimensional photonic crystals

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**Abstract**

The paper presents a comparison between experimental and theoretical transmission curves obtained for a two-dimensional dielectric photonic crystal. We get a perfect agreement on the transmission peaks inside the gap for a crystal with microcavities. Some discrepancies observed on the low frequency side of the gap are analyzed, and we highlight the most probable origins of this disagreement. © 1999 Elsevier Science B.V. All rights reserved.

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## 1. Introduction

Due to the periodicity of the structure, photonic crystals exhibit photonic band gaps. Inside these gaps, photons cannot propagate in a given range of wavelengths, whatever the direction of propagation [1–3]. By introducing defects inside the crystal, it is possible to generate localized resonances. For example, microcavities created inside the crystal can play the role of relays for the photons at wavelengths inside the gap. As a consequence, if the resonant wavelength of a microcavity lies inside the gap of the periodic crystal, peaks of transmission can occur [2,4].

The aim of this paper is to present a comparison between experimental data obtained for a dielectric photonic crystal in the microwave region between 2 and 6 GHz and theoretical results obtained from a rigorous electromagnetic method. Such comparisons have been published in the recent years. Sigalas et al. [5] have compared their numerical computations with experimental results obtained by Smith et al. [6] around 10 GHz. It is to be noticed that in Ref. [6], the experimental results are obtained by placing a 2D photonic crystal inside a waveguide. In that case, the theoretical data are deduced from the supercell hypothesis where the microcavity has been periodized, in such a way that the photonic crystal remains periodic. Gadot et al. [7] have presented comparisons between experimental results in the range 30–75 GHz and theoretical results obtained from the transfer matrix method developed by Pendry and McKinnon [8]. The same transfer matrix method was used by Ozbay et al. [9] at frequencies close to 100 GHz. In Refs. [7,9], no comparison between experiment and theory has been presented for crystals doped with microcavities.

## 2. Materials and methods

The interest of our theoretical method lies in the fact that it is able to deal with a crystal containing a finite number of rods [10]. In this theory, the rods are of infinite length, and each rod is characterized by its scattering matrix which links the diffracted field to the incoming one, these fields being represented by Fourier–Bessel expansions. From translation properties of Bessel functions, the

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scattering problem is reduced to the resolution of a linear system of equations. This method combines very good relative accuracy (better than 1%) with rapid computation. The number of rods can reach 500 on an ordinary desktop computer with 256 M of RAM. This theoretical method allows us to model a crystal under conditions that are very close to the actual experimental device, without supercell hypothesis.

The photonic crystal is shown in Figs. 1 and 2. This crystal of hexagonal symmetry is made of 80 dielectric rods. These rods are molded with a composite material made by IMO Company (91360 Epinay sur Orge, France), containing alumina and synthetic resin. The distance between the centers of two neighboring rods is 33 mm. The diameter and the length of the rods are 10 mm and 300 mm, respectively, and their permittivity was given to be equal to $6 \pm 2$ by the manufacturer. Fig. 3 shows the experimental setup. The crystal is placed between two horns (ARA DRG 118A). It is possible to adjust the distances $d_e$ and $d_r$ between the crystal and the emitting or receiving horns, respectively. The experimental data reported in the following have been obtained by averaging the transmission measured for different values of $(d_e, d_r)$, $d_e$ and $d_r$ being chosen in the range 4–12 cm, in order to reduce the effects of interaction between the horns and the crystal. In order to reduce external disturbances, the device is surrounded by absorbent foam. The emitting horn is fed by a sweeper (HP83640). Reception and analysis of the signal are performed by a vectorial network analyzer (HP8510C) and a test set (HP8516). The frequency of the electromagnetic signal can be varied in the range 2–6 GHz. The dynamic range of the experimental setup can be evaluated to 70 dB.

In the theoretical results, the crystal is illuminated by a plane wave and the transmission is computed in the following way. We compute the flux $\Phi_t$ of the Poynting vector on a line lying on the exit side of the crystal. This line (do not forget that it is a two-dimensional problem) represents the entry face of the receiving horn. In the following, this line is situated 8 cm from the centers of the exit side rods and is 20 cm long. We also compute on the same line the flux $\Phi_i$ of the Poynting vector of the incident wave only (in other words, without the crystal). The transmission is defined as the ratio $\Phi_t / \Phi_i$.

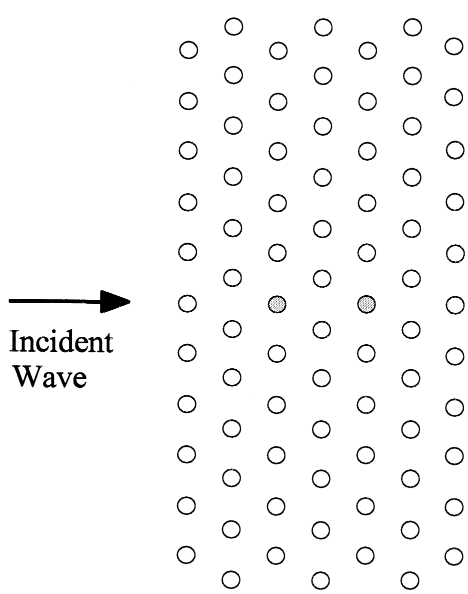


Fig. 1. Scheme of the photonic crystal. In the non-doped crystal, all the rods are present. The doped crystal is obtained by suppressing the two grey rods.

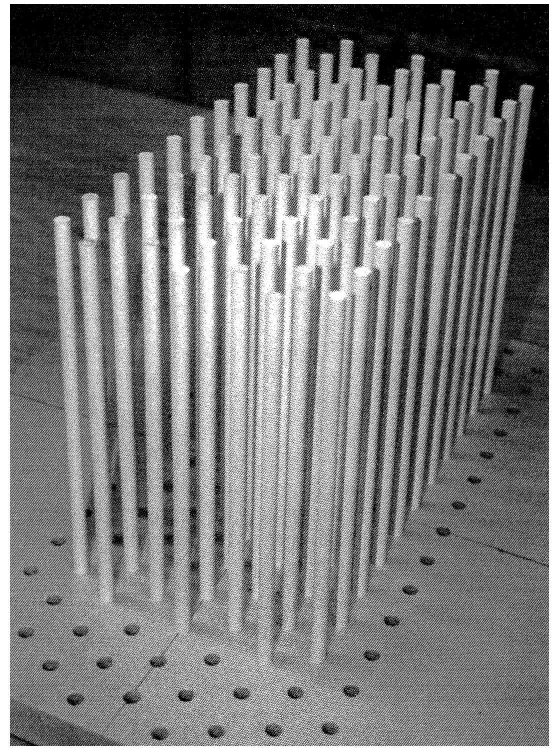

Fig. 2. Non-doped crystal realization.

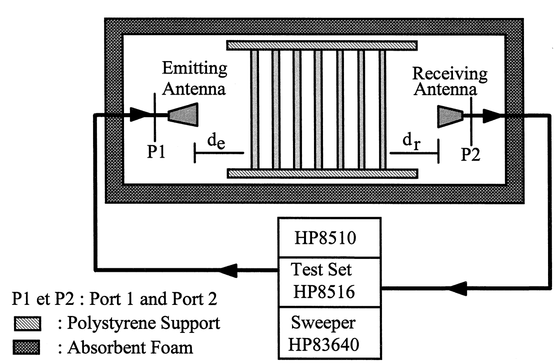


Fig. 3. Experimental setup.

## 3. Experimental

The discrepancy between experimental data and our preliminary theoretical results on both sides of the gap led us to perform precise measurements of the permittivity of the rods. We used a classical technique of guided propagation by placing a sample of dielectric material of rod inside a coaxial cell. The measurements have shown that the material is lossless and its relative magnetic constant is equal to unity. Furthermore, we found that the rods are not all identical from the electromagnetic point of view. Nevertheless, their permittivities stay in the range $6 \pm 0.3$ for frequencies between 2 and 6 GHz.

Fig. 4 shows the experimental and theoretical transmission curves obtained for the non-doped crystal of Fig. 1. The same curves are given in Fig. 5 for the doped crystal. The theoretical data have been obtained by using a permittivity equal to 6. The transmission gap extends from 3.5 to 5 GHz. For the doped crystal, two transmission peaks appear inside the gap at 4.08 GHz and 4.30 GHz (experimental results), or 4.09 GHz and 4.33 GHz (theoretical results). The origin of the double peaks is the existence of a couple of resonance frequencies for the microcavity made by the two defects, as shown in Ref. [4]. A very good agreement between experimental and theoretical data is obtained for the location of the peaks. On the other hand, significant discrepancies occur on both sides of the gap and mostly on the left-hand side, the right-hand side of the gap being located at 5 GHz precisely in both cases.

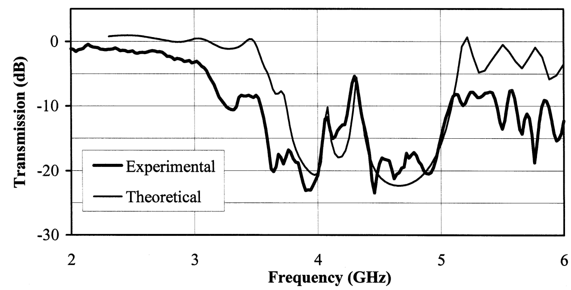


Fig. 5. Same as in Fig. 4, but for the doped crystal.

## 4. Results and discussion

The agreement of the location of the peaks is not surprising. Indeed, these peaks are caused by resonance phenomena inside the microcavities. The resonance frequency is independent of many experimental parameters such as the width and the incidence of the light beam. In contrast, the transmission factor strongly depends on these parameters. The fact that the main discrepancies appear on both sides of the gap could be caused by many reasons. First, it has been mentioned that the permittivities of the rods are not equal. Fig. 6 shows three theoretical transmission curves obtained by assuming $\varepsilon = 5.7$, 6.0 and 6.3 for all the rods of the crystal of Fig. 1. Obviously, the effects of the variation of permittivity appear to be greater on the left-hand side of the gap. We could be inclined to deduce that the dispersion of the permittivity causes the discrepancy observed in Figs. 4 and 5. However, additional calculations made by considering a random permittivity of the rods in the range 5.7–6.3 for all the rods show that the effect of a random dispersion of the rods is much smaller than the effect of a general change of permittivity of all rods. In practice, this curve is almost identical to the curve corresponding to $\varepsilon = 6$ in Fig. 6. Thus, it seems that the dispersion of permittivity is not the main reason for the discrepancy observed in Figs. 4 and 5.

A second explanation could be found in the property of ultrarefractivity proposed by specialists of photonic crystals [11]. From diagrams of dispersion of light velocity inside the crystal, these specialists predict that at the edge of a gap, a photonic crystal can behave as a homogeneous material with a permittivity close to zero. A finite size incident beam cannot be rigorously parallel; thus, it contains directions of propagation slightly different from the average direction. When the crystal is removed, this small angular range of the direction of propagation has no effect in practice on experimental measurements. However, if we conjecture that the crystal simulates a homogeneous mate-

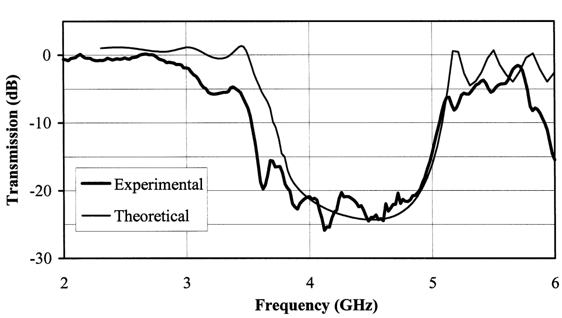


Fig. 4. Transmission curves for the non-doped crystal.

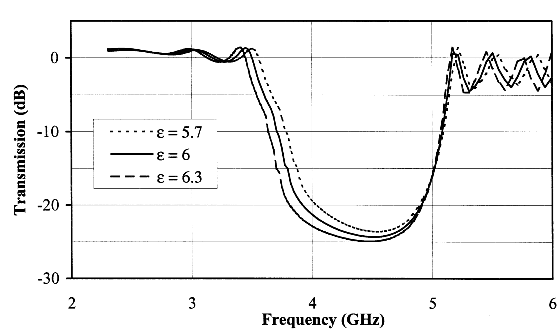


Fig. 6. Non-doped crystal: theoretical results for different values of the permittivity $\varepsilon$.

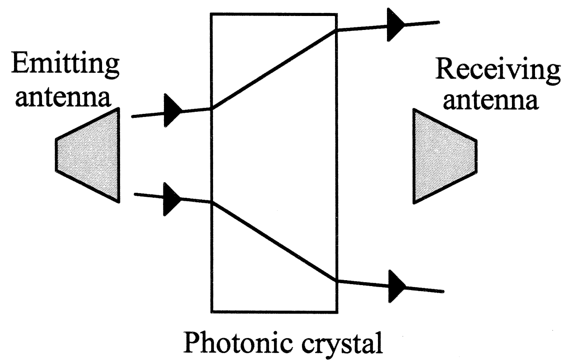


Fig. 7. Transmission through the crystal according to the ultrarefraction conjecture.

rial with permittivity (thus optical index) close to zero, this narrow angular range may generate a significant widening of the outgoing beam, thus a reduction of the measured transmission (see Fig. 7). Indeed, in Figs. 4 and 5, the main discrepancies are observed on both sides of the gap (it is worth noticing that a second gap, which cannot be seen in Figs. 4 and 5, begins at a frequency close to 6 GHz) and always consist of a reduction of the transmission factor. Of course, this reasoning is not a proof that ultrarefraction is the actual explanation of the discrepancies observed between experimental and theoretical data. The existence of ultrarefraction phenomena will be shown from electromagnetic theory in a following paper. In this study, it emerges that these phenomena are very selective in wavelength and thus, they cannot explain entirely the discrepancies observed in Figs. 4 and 5.

Other explanations of the discrepancies can be found. In our calculations, the incident wave is a plane wave of infinite extension, and the crystal is made with rods of infinite length. In practice, at the lower frequency of the gap, the length of the rods is of the order of three wavelengths only. Moreover, the presence of the emitting and receiving horns close to the crystal gives rise to complicated interaction phenomena which are not taken into account in our calculations. The support (polystyrene) of the crystal could also have an influence on the measured transmission.

## 5. Conclusion

Comparisons between experimental and theoretical data for transmission of doped and non-doped dielectric photonic crystals in the microwave region have shown a good agreement, especially in the location of transmission peaks for doped crystal. On both sides of the gap, measured transmission factors can be significantly smaller than theoretical ones. Ultrarefractive phenomena could be an explanation of this difference.

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