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Symmetry relations for reflection and transmission coefficients of magneto-optic systems

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Abstract

In magneto-optic systems, Kerr and Faraday effects bring into play reflection and transmission matrices. Considering planar multilayer systems, we show that the elements of the reflection and transmission matrices obey general and simple relations valid whatever the geometry of the magneto-optic device may be. © 2002 Elsevier Science B.V. All rights reserved.

1. Introduction

Magneto-optic effects stems from the interaction of light with media submitted to a magnetic field. This interaction gives rise to a rotation of the polarization vector of both the transmitted and reflected fields called the Faraday and Kerr effects [1–3], respectively. Thus when a magneto-optic system is illuminated by a TE or s (TM or p) polarized light, the reflected and transmitted fields include a p (s) component. Consequently, Kerr and Faraday effects involve reflection and transmission matrices:

$$[r] = \begin{bmatrix} r^{ss} & r^{sp} \\ r^{ps} & r^{pp} \end{bmatrix}, \quad [t] = \begin{bmatrix} t^{ss} & t^{sp} \\ t^{ps} & t^{pp} \end{bmatrix},$$

where the right-hand superscript denotes the incident polarization.

Several theories have been developed for the determination of the elements of the $[r]$ and $[t]$ matrices [4–9]. For polar and longitudinal configurations [10,11], the calculations [4–9] always show that $r^{sp} = \pm r^{ps}$ whatever the system under consideration may be. Thus, it is tempting to wonder whether such a result would be a general property of magneto-optic devices. In this paper, we show that this is indeed the case. We also address the question of the link between t^{sp} and t^{ps}. The theory that we have developed is presented in Sections 2.1–2.4. Properties concerning the diagonal elements of $[t]$ are considered in Section 2.5. The demonstrations of these relations are based on the use of the Lorentz reciprocity theorem [12] and

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applies to any planar multilayer and/or gradient permittivity magneto-optic system. Section 3 is devoted to numerical calculations which allow one to check the general relations derived in Section 2.

2. Theory

2.1. General considerations

This section is devoted to the constitutive relations, to Maxwell equations and to the linearity of the magneto-optic system.

The system under consideration is depicted in Fig. 1 and consists of three domains a, b, and c. The plane of incidence is the plane (xOy) and Fig. 1 corresponds to the two-dimensional situation for which $\partial/\partial z = 0$. The incident plane wave is s or p polarized i.e. electric or magnetic field parallel to the z-axis respectively. The superstrate a $(y > 0)$ and substrate b $(y < -e)$ are assumed to be homogeneous, isotropic and non-magnetic. Medium c exhibits magneto-optic effects and may include one or several magneto-optic layers. For the sake of generality, its permittivity tensor is y-dependent.

At optical frequencies, it is possible to describe a magneto-optic medium by its permittivity tensor $[\varepsilon]$ and a permeability being that of vacuum [10,11,13]. Thus:

$$\vec{D} = \varepsilon_0[\varepsilon]\vec{E}, \tag{1a}$$

$$\vec{B} = \mu_0\vec{H} \tag{1b}$$

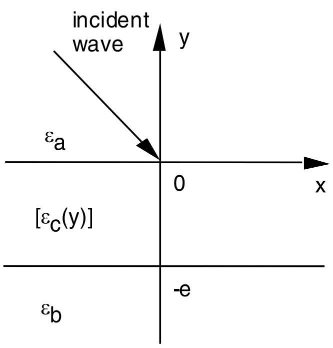

Fig. 1. An example of a three domain system including a magneto-optic layer with permittivity tensor $[\varepsilon_c(y)]$.

and Maxwell equations are written as:

$$\text{curl } \vec{E} = +i\omega\mu_0\vec{H}, \tag{2a}$$

$$\text{curl } \vec{H} = -i\omega\varepsilon_0[\varepsilon]\vec{E}, \tag{2b}$$

where an $e^{-i\omega t}$ time-dependence is understood. The permittivity $[\varepsilon]$ in Eqs. (1a) and (2b) depends on y as follows:

$$[\varepsilon(y)] = \begin{cases} \varepsilon_a & \text{for } y > 0, \\ [\varepsilon_c(y)] & \text{for } -e < y < 0, \\ \varepsilon_b & \text{for } y < -e. \end{cases} \tag{3a}$$

The dielectric tensor $[\varepsilon_c]$ depends on the magneto-optical orientation. The three fundamental situations are the following:

- the polar magneto-optical orientation for which,

$$\left[\varepsilon_c^{pol}\right] = \begin{bmatrix} \varepsilon_d & 0 & \varepsilon_{nd} \\ 0 & \varepsilon_d & 0 \\ -\varepsilon_{nd} & 0 & \varepsilon_d \end{bmatrix}, \tag{3b}$$

- the longitudinal magneto-optical orientation for which,

$$[\varepsilon_c^{long}] = \begin{bmatrix} \varepsilon_d & 0 & 0 \\ 0 & \varepsilon_d & \varepsilon_{nd} \\ 0 & -\varepsilon_{nd} & \varepsilon_d \end{bmatrix}, \tag{3c}$$

- the transverse magneto-optical orientation for which,

$$[\varepsilon_c^{transv}] = \begin{bmatrix} \varepsilon_d & \varepsilon_{nd} & 0 \\ -\varepsilon_{nd} & \varepsilon_d & 0 \\ 0 & 0 & \varepsilon_d \end{bmatrix}. \tag{3d}$$

The dielectric tensor $[\varepsilon_c]$ in Eqs. (3a)–(3d) may be rewritten as:

$$[\varepsilon_c] = [\varepsilon_d] + [\varepsilon_{nd}], \tag{4}$$

where $[\varepsilon_d]$ and $[\varepsilon_{nd}]$ are the symmetric and antisymmetric parts of $[\varepsilon_c]$, respectively. The symmetric part does not give rise to the Faraday or Kerr effects [9]. Thus we assumed in Eqs. (3b)–(3d) that medium c is isotropic when no magnetic field is applied.

Let us now show that the problem at hand is a linear one with respect to the magneto-optic effect.

Eqs. (2a), (2b) and (4), written in medium c, lead to (see Appendix A):

$$\Delta\vec{E} + k_0^2\varepsilon_d\vec{E} + \mathrm{grad}\left(E_y\frac{1}{\varepsilon_d}\frac{d\varepsilon_d}{dy}\right) = -\mathrm{grad}\left(\frac{1}{\varepsilon_d}\mathrm{div}\left([\varepsilon_{nd}]\vec{E}\right)\right) - k_0^2[\varepsilon_{nd}]\vec{E} \tag{5a}$$

with

$$k_0 = \frac{\omega}{c}. \tag{5b}$$

From Eqs. (5a) and (3d), taking into account that $\partial/\partial z = 0$, it is found that the transverse magneto-optical orientation does not induce any rotation of the polarization vector.

The antisymmetric part of $[\varepsilon_c]$ induces a small rotation of the plane of polarisation [10]. Therefore:

$$[\varepsilon_{nd}]\vec{E} \approx [\varepsilon_{nd}]\vec{E}_d, \tag{5c}$$

where $\vec{E}_d$ denotes the electric field only due to the symmetric part of $[\varepsilon_c]$. Thus Eq. (5a) writes:

$$\Delta\vec{E} + k_0^2\varepsilon_d\vec{E} + \mathrm{grad}\left(E_y\frac{1}{\varepsilon_d}\frac{d\varepsilon_d}{dy}\right) \approx -\mathrm{grad}\left(\frac{1}{\varepsilon_d}\mathrm{div}\left([\varepsilon_{nd}]\vec{E}_d\right)\right) - k_0^2[\varepsilon_{nd}]\vec{E}_d. \tag{5d}$$

Eq. (5d) is a linear one. If the magneto-optical orientation is in the plane of incidence xOy then:

$$[\varepsilon_{nd}] = [\varepsilon_{nd}^{long}] + [\varepsilon_{nd}^{pol}]. \tag{6a}$$

Eqs. (5d) and (6a) show that, in this case, the response in reflection (Kerr effect) or transmission (Faraday effect) will be the sum of the responses corresponding to each of the situations Eqs. (3b) and (3c). Therefore in this paper, we only consider longitudinal and polar magneto-optical orientations.

Moreover, as will be seen in the following section, the transposed permittivity tensor denoted ${}^t[\varepsilon]$ will be needed. According to Eqs. (3a)–(3d):

$${}^t[\varepsilon_c] = [\varepsilon_c(-\varepsilon_{nd})]. \tag{6b}$$

Because of the linearity of Eq. (5d), changing ε_{nd} to $-\varepsilon_{nd}$ will not modify the absolute value of the off-diagonal elements of $[r]$ and $[t]$, only the signs of these elements will be changed.

2.2. The Lorentz reciprocity theorem

Let us consider two solutions of the Maxwell equations at the same frequency ω, labeled 1 and 2, without any source term; the Lorentz reciprocity theorem [12] states that:

$$\mathrm{div}\left(\vec{E}_1 \times \vec{H}_2 - \vec{E}_2 \times \vec{H}_1\right) = 0. \tag{7}$$

When the system includes media with nonsymmetrical permittivity tensors, which is the case here, solution 1 is associated to the permittivity tensor $[\varepsilon]$ whereas solution 2 has to be associated to the transposed permittivity tensor ${}^t[\varepsilon]$. To keep its generality to the calculation, we do not specify for the moment the nature of solutions 1 and 2.

Let us consider Fig. 2: it is characterized by one incident wave (labeled i) in media a and b, each incident wave giving rise to a reflected and a transmitted wave. Therefore the total electromagnetic (EM) field in each outside medium is the sum of the incident, reflected and transmitted waves. The angles of incidence θ_a and θ_b in media a and b, respectively, are chosen such that the reflected and transmitted EM fields in these media propagate in the same direction. This requires:

$$n_a \sin\theta_a = n_b \sin\theta_b, \tag{8}$$

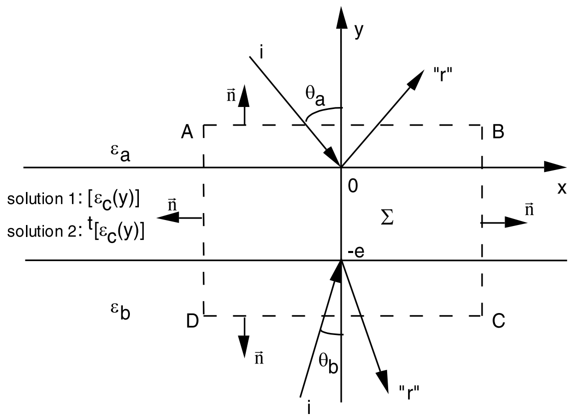

Fig. 2. The geometry and the contour used for deriving the relations between the off-diagonal elements of $[r]$ and $[t]$. Solution 1: both incident waves are p-polarized and $[\varepsilon(-e < y < 0)] = [\varepsilon_c]$. Solution 2: both incident waves are s-polarized and $[\varepsilon(-e < y < 0)] = {}^t[\varepsilon_c]$.

where n_a and n_b are the indices of refraction of medium a and b, respectively.

Eq. (8) is fulfilled throughout this paper; "r" denotes the outgoing field which is the sum of the reflected and transmitted fields in superstrate and substrate.

Integrating Eq. (7) in a volume V yields:

$$\int\int_V\int \mathrm{div}(\vec{E}_1 \times \vec{H}_2 - \vec{E}_2 \times \vec{H}_1)\,\mathrm{d}V = 0. \tag{9a}$$

The system being z-independent, the integration is performed on the surface Σ illustrated in Fig. 2 (instead of volume V). Thus:

$$\int\int_\Sigma \mathrm{div}(\vec{E}_1 \times \vec{H}_2 - \vec{E}_2 \times \vec{H}_1)\,\mathrm{d}x\,\mathrm{d}y = 0 \tag{9b}$$

Use of the two-dimensional form of the divergent theorem allows us to rewrite Eq. (9b) as:

$$\oint_{C=ABCD} \left(\vec{E}_1 \times \vec{H}_2 - \vec{E}_2 \times \vec{H}_1\right) \cdot \vec{n}\,\mathrm{d}\ell = 0, \tag{9c}$$

where $\vec{n}$ is the unit vector, in the xOy plane, perpendicular to the boundary C (called $ABCD$) of surface Σ and pointing toward the outside of Σ (see Fig. 2) and ℓ is a curvilinear abscissae.

The length of $AB = DC = d$ is chosen in such a way that the total EM field has the same value along AD and BC. Consequently Eq. (9c) reduces to:

$$\int_0^{\mathrm{d}} \Big\{(\vec{E}_1 \times \vec{H}_2 - \vec{E}_2 \times \vec{H}_1)|_{y=0} \cdot \vec{n}_y - (\vec{E}_1 \times \vec{H}_2 - \vec{E}_2 \times \vec{H}_1)|_{y=-e} \cdot \vec{n}_y\Big\}\mathrm{d}x = 0, \tag{10a}$$

with $\vec{n}_y = (0, 1, 0)$.

From Eq. (8), it follows that the EM field in all regions have the same $e^{i\alpha x}$-dependence with:

$$\alpha = (\omega/c)n_a \sin\theta_a = (\omega/c)n_b \sin\theta_b. \tag{10b}$$

Let us define:

$$\vec{Q}^a = \left(\vec{E}_1 \times \vec{H}_2 - \vec{E}_2 \times \vec{H}_1\right)\Big|_{y=0}, \tag{10c}$$

$$\vec{Q}^b = \left(\vec{E}_1 \times \vec{H}_2 - \vec{E}_2 \times \vec{H}_1\right)\Big|_{y=-e}, \tag{10d}$$

$$\vec{Q} = \vec{Q}^a - \vec{Q}^b, \tag{11a}$$

Then Eqs. (10a) leads to:

$$\vec{Q} \cdot \vec{n}_y = 0, \tag{11b}$$

The remaining of the calculation proceeds as follows: we first derive the expression of $\vec{Q}^a$. Then a first couple of solutions 1 and 2 is specified. For this set, we get the expression of the scalar product $\vec{Q}.\vec{n}_y$. This allows us to derive the relations between the off-diagonal elements of matrices $[r]$ and $[t]$. A second couple of solutions 1 and 2 yields relations which involve the diagonal elements of $[t]$.

2.3. Calculation of $\vec{Q}^a$

$\vec{Q}^a$ is rewritten as:

$$\vec{Q}^a = (\vec{E}_1^a \times \vec{H}_2^a - \vec{E}_2^a \times \vec{H}_1^a). \tag{12a}$$

$$\vec{Q}^a = \Big[(\vec{E}^a_{1,i} + \vec{E}^a_{1,\text{“r”}}) \times (\vec{H}^a_{2,i} + \vec{H}^a_{2,\text{“r”}}) - (\vec{E}^a_{2,i} + \vec{E}^a_{2,\text{“r”}}) \times (\vec{H}^a_{1,i} + \vec{H}^a_{1,\text{“r”}})\Big]. \tag{12b}$$

In the outside media the EM field is a sum of plane waves; for each of them, the following relations hold:

$$\vec{H} = \frac{1}{\omega\mu_0}\vec{k} \times \vec{E}, \tag{13a}$$

$$\vec{k} \cdot \vec{E} = 0, \tag{13b}$$

where $\vec{k}$ denotes the wavevector.

Use of Eqs. (13a) and (13b) shows that Eq. (12b) becomes:

$$\vec{Q}^a = \vec{E}^a_{1,i} \times \vec{H}^a_{2,\text{“r”}} - \vec{E}^a_{2,i} \times \vec{H}^a_{1,\text{“r”}} + \vec{E}^a_{1,\text{“r”}} \times \vec{H}^a_{2,i} - \vec{E}^a_{2,\text{“r”}} \times \vec{H}^a_{1,i}. \tag{14a}$$

In deriving Eq. (14a) the following relations have been used:

$$\vec{E}^a_{1,i} \cdot \vec{k}_{a,i} = 0, \tag{14b}$$

$$\vec{E}^a_{2,i} \cdot \vec{k}_{a,i} = 0, \tag{14c}$$

$$\vec{E}^a_{1,\text{“r”}} \cdot \vec{k}_{a,r} = 0, \tag{14d}$$

$$\vec{E}^a_{2,\text{“r”}} \cdot \vec{k}_{a,r} = 0. \tag{14e}$$

where $\vec{k}_{a,i}$, $\vec{k}_{a,r}$ denote the wavevectors of the incident and "reflected" waves in medium a.

2.4. The first couple of solutions 1 and 2

In Fig. 2, solution 1 corresponds to incident p polarized plane waves in media a and b together with the permittivities ε_a, ε_b and $[\varepsilon_c]$ whereas solution 2 is associated to incident s polarized plane waves in media a and b together with the permittivities ε_a, ε_b and ${}^t[\varepsilon]$.

Making explicit the p and s polarization of the incident waves for solutions 1 and 2 leads to:

$$\vec{Q}^{a} = \vec{Q}^{a,p} - \vec{Q}^{a,s}, \tag{15a}$$

with

$$\vec{Q}^{a,p} = \vec{E}^{a,p}_{1,i} \times \vec{H}^{a}_{2,\text{“r”}} - \vec{E}^{a}_{2,\text{“r”}} \times \vec{H}^{a,p}_{1,i}, \tag{15b}$$

$$\vec{Q}^{a,s} = \vec{E}^{a,s}_{2,i} \times \vec{H}^{a}_{1,\text{“r”}} - \vec{E}^{a}_{1,\text{“r”}} \times \vec{H}^{a,s}_{2,i}. \tag{15c}$$

Let us calculate $\vec{Q}^{a,p}$. Eqs. (13a) and (13b) implies that:

$$\vec{E} = -\frac{\omega\mu_0}{k^2}\vec{k} \times \vec{H}. \tag{16}$$

According to Eqs. (16) and (15b), we get:

$$\begin{aligned}\frac{k_a^2}{\omega\mu_0}\vec{Q}^{a,p}\cdot\vec{n}_y &= (\vec{H}^{a,p}_{1,i}\cdot\vec{H}^{a}_{2,\text{“r”}})(\vec{k}_{a,i}\cdot\vec{n}_y)\\ &\quad - (\vec{H}^{a}_{2,\text{“r”}}\cdot\vec{k}_{a,i})(\vec{H}^{a,p}_{1,i}\cdot\vec{n}_y)\\ &\quad - (\vec{H}^{a,p}_{1,i}\cdot\vec{H}^{a}_{2,\text{“r”}})(\vec{k}_{a,r}\cdot\vec{n}_y)\\ &\quad + (\vec{H}^{a,p}_{1,i}\cdot\vec{k}_{a,r})(\vec{H}^{a}_{2,\text{“r”}}\cdot\vec{n}_y),\end{aligned} \tag{17}$$

where $k_a = (\omega/c)\sqrt{\varepsilon_a}$.

In Eq. (17), one has:

$$\vec{H}^{a,p}_{1,i}\cdot\vec{n}_y = 0, \tag{18a}$$

$$\vec{H}^{a,p}_{1,i}\cdot\vec{k}_{a,r} = 0, \tag{18b}$$

because $\vec{H}^{a,p}_{1,i}$ is along z and $\vec{k}_{a,r}$ is in the plane of incidence (xOy). Thus

$$\vec{Q}^{a,p}\cdot\vec{n}_y = \frac{2\omega\mu_0}{k_a^2}[\vec{H}^{a,p}_{1,i}\cdot\vec{H}^{a,p}_{2,\text{“r”}}](\vec{k}_{a,i}\cdot\vec{n}_y), \tag{19}$$

where use has been made of the fact that:

$$\vec{H}^{a,p}_{1,i}\cdot\vec{H}^{a}_{2,\text{“r”}} = \vec{H}^{a,p}_{1,i}\cdot\vec{H}^{a,p}_{2,\text{“r”}}, \tag{20a}$$

$$\vec{k}_{a,r}\cdot\vec{n}_y = -\vec{k}_{a,i}\cdot\vec{n}_y. \tag{20b}$$

Maxwell Eqs. (2a) and (2b) are invariant in the transformation $\vec{E} \leftrightarrow \vec{H}$, $\varepsilon_0[\varepsilon] \leftrightarrow -\mu_0$, we thus obtain (together with the substitution $1 \leftrightarrow 2$ and $p \rightarrow s$):

$$\vec{Q}^{a,s}\cdot\vec{n}_y = -\frac{2}{\omega\mu_0}[\vec{E}^{a,s}_{2,i}\cdot\vec{E}^{a,s}_{1,\text{“r”}}](\vec{k}_{a,i}\cdot\vec{n}_y). \tag{21}$$

Eq. (21) can also be obtained by a direct calculation following the same lines as those that lead to Eq. (19).

Finally we get (from Eqs. (15a), (19), (21), (11a)):

$$\begin{aligned}\vec{Q}\cdot\vec{n}_y = &\left[\frac{2\omega\mu_0}{k_a^2}[\vec{H}^{a,p}_{1,i}\cdot\vec{H}^{a,p}_{2,\text{“r”}}]\right.\\ &\left.+\frac{2}{\omega\mu_0}[\vec{E}^{a,s}_{2,i}\cdot\vec{E}^{a,s}_{1,\text{“r”}}]\right](\vec{k}_{a,i}\cdot\vec{n}_y)\\ &-\left[\frac{2\omega\mu_0}{k_b^2}[\vec{H}^{b,p}_{1,i}\cdot\vec{H}^{b,p}_{2,\text{“r”}}]\right.\\ &\left.+\frac{2}{\omega\mu_0}[\vec{E}^{b,s}_{2,i}\cdot\vec{E}^{b,s}_{1,\text{“r”}}]\right](\vec{k}_{b,i}\cdot\vec{n}_y).\end{aligned} \tag{22a}$$

As is well known the magneto-optic effect induces a rotation of the polarization leading for the "r" wave:

- to a p component for solution 2 (which corresponds to s-polarized incident waves),
- to a s component for solution 1 (which corresponds to p-polarized incident waves).

In Eq. (22a) this effect is expressed by the scalar products involving the electric and magnetic fields. The magnetic field of the p-polarized "r"-wave is along z but it can point either in the same or in the opposite direction as that of the incident magnetic field. This depends on the magneto-optic case, polar or longitudinal, which is considered. The same conclusion applies to the sense of the electric field of the s-polarized "r"-wave with respect to that of the electric field of the s-incident wave. Thus we introduce the notations

$$c_h^a = \pm, \quad c_e^a = \pm, \quad c_h^b = \pm \quad \text{and } c_e^b = \pm. \tag{22b}$$

It is important to realize that the $\pm$ signs are *not* correlated. Then Eq. (22a) writes:

$$\vec{Q}\cdot\vec{n}_y = \left[c_{\rm h}^{\rm a}\frac{2\omega\mu_0}{k_{\rm a}^2}H_{1,{\rm i}}^{\rm a,p}|H_{2,\text{``r''}}^{\rm a,p}| + c_{\rm e}^{\rm a}\frac{2}{\omega\mu_0}E_{2,{\rm i}}^{\rm a,s}|E_{1,\text{``r''}}^{\rm a,s}|\right](\vec{k}_{\rm a,i}\cdot\vec{n}_y) - \left[c_{\rm h}^{\rm b}\frac{2\omega\mu_0}{k_{\rm b}^2}H_{1,{\rm i}}^{\rm b,p}|H_{2,\text{``r''}}^{\rm b,p}| + c_{\rm e}^{\rm b}\frac{2}{\omega\mu_0}E_{2,{\rm i}}^{\rm b,s}|E_{1,\text{``r''}}^{\rm b,s}|\right](\vec{k}_{\rm b,i}\cdot\vec{n}_y). \tag{23a}$$

It can be assumed, without loss of generality, that the incident magnetic and electric fields are positive. This has been done for the derivation of Eq. (23a).

Finally use of Eqs. (13a) and (13b) yields:

$$\vec{Q}\cdot\vec{n}_y = \frac{2}{\omega\mu_0}\left\{\left[c_{\rm h}^{\rm a}E_{1,{\rm i}}^{\rm a,p}|E_{2,\text{``r''}}^{\rm a,p}| + c_{\rm e}^{\rm a}E_{2,{\rm i}}^{\rm a,s}|E_{1,\text{``r''}}^{\rm a,s}|\right](\vec{k}_{\rm a,i}\cdot\vec{n}_y) - \left[c_{\rm h}^{\rm b}E_{1,{\rm i}}^{\rm b,p}|E_{2,\text{``r''}}^{\rm b,p}| + c_{\rm e}^{\rm b}E_{2,{\rm i}}^{\rm b,s}|E_{1,\text{``r''}}^{\rm b,s}|\right](\vec{k}_{\rm b,i}\cdot\vec{n}_y)\right\}. \tag{23b}$$

It is understood that in Eq. (23b) the electric field with superscript a, b are calculated at $y = 0, -e$ respectively (see Eqs. (10c) and (10d)).

We remind the reader that in Eq. (23b) the "r"-wave is a sum of a reflected and transmitted wave (subscript r and t respectively). For example:

$$E_{2,\text{``r''}}^{\rm a,p} = E_{2,{\rm r}}^{\rm a,p} + E_{2,{\rm t}}^{\rm a,p}. \tag{24}$$

To pursue the calculation it is necessary to define the elements of the $[r]$ and $[t]$ matrices. For the time being, only the off diagonal elements are needed:

$$|E_{2,{\rm r}}^{\rm a,p}(y=0)| = |r_{\rm a}^{\rm ps}({}^{\rm t}[\varepsilon])|E_{2,{\rm i}}^{\rm a,s}(y=0), \tag{25a}$$

$$|E_{1,{\rm r}}^{\rm a,s}(y=0)| = |r_{\rm a}^{\rm sp}([\varepsilon])|E_{1,{\rm i}}^{\rm a,p}(y=0), \tag{25b}$$

$$|E_{2,{\rm r}}^{\rm b,p}(y=-e)| = |r_{\rm b}^{\rm ps}({}^{\rm t}[\varepsilon])|E_{2,{\rm i}}^{\rm b,s}(y=-e), \tag{25c}$$

$$|E_{1,{\rm r}}^{\rm b,s}(y=-e)| = |r_{\rm b}^{\rm sp}([\varepsilon])|E_{1,{\rm i}}^{\rm b,p}(y=-e), \tag{25d}$$

$$|E_{2,{\rm t}}^{\rm a,p}(y=0)| = |t_{\rm ba}^{\rm ps}({}^{\rm t}[\varepsilon])|E_{2,{\rm i}}^{\rm b,s}(y=-e), \tag{25e}$$

$$|E_{1,{\rm t}}^{\rm a,s}(y=0)| = |t_{\rm ba}^{\rm sp}([\varepsilon])|E_{1,{\rm i}}^{\rm b,p}(y=-e), \tag{25f}$$

$$|E_{2,{\rm t}}^{\rm b,p}(y=-e)| = \left|t_{\rm ab}^{\rm ps}({}^{\rm t}[\varepsilon])\right|E_{2,{\rm i}}^{\rm a,s}(y=0), \tag{25g}$$

$$|E_{1,{\rm t}}^{\rm b,s}(y=-e)| = |t_{\rm ab}^{\rm sp}([\varepsilon])|E_{1,{\rm i}}^{\rm a,p}(y=0). \tag{25h}$$

In this section, we have obtained the expression (Eq. (23b)) of the integrand $\vec{Q}\cdot\vec{n}_y$ occuring in Eq. (10a). Using Eq. (23b) together with Eqs. (25a)–(25h), we now derive the relations between the off-diagonal elements of the $[r]$- and $[t]$-matrices.

2.4.1. Kerr effect: relation between r^{sp} and r^{ps}

Since one deals with a linear problem, $[r]$ and $[t]$ do not depend on the amplitude of the incident waves. Thus we first consider the situation where there is no incident wave in medium b. In this case Eq. (23b) becomes:

$$\vec{Q}\cdot\vec{n}_y = \frac{2}{\omega\mu_0}\left[c_{\rm h}^{\rm a}E_{1,{\rm i}}^{\rm a,p}|E_{2,{\rm r}}^{\rm a,p}| + c_{\rm e}^{\rm a}E_{2,{\rm i}}^{\rm a,s}|E_{1,{\rm r}}^{\rm a,s}|\right](\vec{k}_{\rm a,i}\cdot\vec{n}_y). \tag{26a}$$

Taking Eqs. (25a) and (25b) into account we get:

$$\vec{Q}\cdot\vec{n}_y = \frac{2}{\omega\mu_0}E_{1,{\rm i}}^{\rm a,p}E_{2,{\rm i}}^{\rm a,s}\left[c_{\rm h}^{\rm a}|r_{\rm a}^{\rm ps}({}^{\rm t}[\varepsilon])| + c_{\rm e}^{\rm a}|r_{\rm a}^{\rm sp}([\varepsilon])|\right]\left(\vec{k}_{\rm a,i}\cdot\vec{n}_y\right). \tag{26b}$$

We know from Eq. (11b) that $\vec{Q}\cdot\vec{n}_y = 0$. Hence:

$$c_{\rm h}^{\rm a}|r_{\rm a}^{\rm ps}({}^{\rm t}[\varepsilon])| = -c_{\rm e}^{\rm a}|r_{\rm a}^{\rm sp}([\varepsilon])|. \tag{27}$$

One has to consider the four possible combinations of + and – signs entering Eq. (27) through the c-coefficients defined in Eq. (22b). It is seen that:

if $c_{\rm h}^{\rm a},\ c_{\rm e}^{\rm a} = +,\ +$ or $c_{\rm h}^{\rm a},\ c_{\rm e}^{\rm a} = -,\ -$:

$$r_{\rm a}^{\rm ps}({}^{\rm t}[\varepsilon]) = r_{\rm a}^{\rm sp}([\varepsilon]) = 0, \tag{28a}$$

and if $c_{\rm h}^{\rm a},\ c_{\rm e}^{\rm a} = +,\ -$ or $c_{\rm h}^{\rm a},\ c_{\rm e}^{\rm a} = -,+$:

$$|r_{\rm a}^{\rm ps}({}^{\rm t}[\varepsilon])| = |r_{\rm a}^{\rm sp}([\varepsilon])|. \tag{28b}$$

Repeating the above mentioned reasoning with the incident wave in medium b instead of medium a leads to the same Eqs. (28a) and (28b) with $a \to b$. So finally either:

$$r^{\rm ps}({}^{\rm t}[\varepsilon]) = r^{\rm sp}([\varepsilon]) = 0, \tag{29a}$$

or

$$|r^{\rm ps}({}^{\rm t}[\varepsilon])| = |r^{\rm sp}([\varepsilon])|. \tag{29b}$$

In the latter case, as a consequence of the remark at the end of Section 2.1, we obtain:

$$|r^{ps}([\varepsilon])| = |r^{sp}([\varepsilon])|, \tag{29c}$$

for the off-diagonal elements of the reflection matrix in medium a or b.

Eq. (29a) corresponds to the situation where there is no rotation of polarization. This occurs at least in two situations:

- when medium c exhibits no magneto-optic effect. Obviously this particular case is included in the theory developed in this paper,
- in the transverse magneto-optic orientation.

Eq. (29c) agrees with Eqs. (54) and (55) of [5]. According to the linearity of the problem (see Section 2.1), in the case of magneto-optical orientations in the plane of incidence, i.e. hybrid polar-longitudinal situation, $r^{ps}({}^t[\varepsilon])$ and $r^{sp}([\varepsilon])$ are linear combinations of off-diagonal elements of $[r]$ corresponding to the polar and longitudinal orientations for which Eq. (29c) applies. Thus in the general case no relation can be found between $r^{ps}([\varepsilon])$ and $r^{sp}([\varepsilon])$. This is in agreement with Eqs. (5) and (7) of [7], with Eqs. (21) and (23) of [8] and with Eq. (7) of [9].

So according to Eqs. (29a)–(29c) the off-diagonal elements of the reflection matrix are either null or have the same absolute value.

2.4.2. Faraday effect: relation between t^{sp} and t^{ps}

As a result of Eqs. (11b), (23b) and (27) written in medium a and b (for medium b let a → b), we get:

$$\frac{2}{\omega\mu_0}\Big\{\Big[c_h^a E_{1,i}^{a,p}|E_{2,t}^{a,p}| + c_e^a E_{2,i}^{a,s}|E_{1,t}^{a,s}|\Big](\vec{k}_{a,i}\cdot\vec{n}_y) - \Big[c_h^b E_{1,i}^{b,p}|E_{2,t}^{b,p}| + c_e^b E_{2,i}^{b,s}|E_{1,t}^{b,s}|\Big](\vec{k}_{b,i}\cdot\vec{n}_y)\Big\} = 0. \tag{30a}$$

Use of Eqs. (25e)–(25h) yields:

$$E_{1,i}^{a,p}E_{2,i}^{b,s}\Big[c_h^a|t_{ba}^{ps}|(\vec{k}_{a,i}\cdot\vec{n}_y) - c_e^b|t_{ab}^{sp}|(\vec{k}_{b,i}\cdot\vec{n}_y)\Big] + E_{1,i}^{b,p}E_{2,i}^{a,s}\Big[c_e^a|t_{ba}^{sp}|(\vec{k}_{a,i}\cdot\vec{n}_y) - c_h^b|t_{ab}^{ps}|(\vec{k}_{b,i}.\vec{n}_y)\Big] = 0. \tag{30b}$$

The argument of linearity, previously invoked, allows looking for solutions corresponding to:

$$E_{1,i}^{b,p} = 0 \text{ or } E_{2,i}^{a,s} = 0 \Rightarrow c_h^a\big|t_{ba}^{ps}({}^t[\varepsilon])\big|k_a\cos\theta_a = -c_e^b\big|t_{ab}^{sp}([\varepsilon])\big|k_b\cos\theta_b, \tag{31a}$$

or to

$$E_{1,i}^{a,p} = 0 \text{ or } E_{2,i}^{b,s} = 0 \Rightarrow c_e^a\big|t_{ba}^{sp}([\varepsilon])\big|k_a\cos\theta_a = -c_h^b\big|t_{ab}^{ps}({}^t[\varepsilon])\big|k_b\cos\theta_b. \tag{31b}$$

Since $k_a\cos\theta_a > 0$ and $k_b\cos\theta_b > 0$, Eq. (31a) gives:

- if $c_h^a, c_e^b = +,+$ or $c_h^a, c_e^b = -,-$:
$$t_{ba}^{ps}({}^t[\varepsilon]) = t_{ab}^{sp}([\varepsilon]) = 0; \tag{32a}$$
- if $c_h^a, c_e^b = +,-$ or $c_h^a, c_e^b = -,+$:
$$|t_{ba}^{ps}({}^t[\varepsilon])|k_a\cos\theta_a = |t_{ab}^{sp}([\varepsilon])|k_b\cos\theta_b. \tag{32b}$$

Eq. (31b) leads to:

- if $c_e^a, c_h^b = +,+$ or $c_e^a,\ c_h^b = -,-$:
$$t_{ba}^{sp}([\varepsilon]) = t_{ab}^{ps}({}^t[\varepsilon]) = 0; \tag{33a}$$
- if $c_e^a,\ c_h^b = +,-$ or $c_e^a, c_h^b = -,+$:
$$|t_{ba}^{sp}([\varepsilon])|k_a\cos\theta_a = |t_{ab}^{ps}({}^t[\varepsilon])|k_b\cos\theta_b. \tag{33b}$$

Eqs. (32a) and (33a) correspond to non-magneto-optic systems or to the transverse magneto-optic orientation. Notice that the choice of signs leading to Eqs. (33a) and (33b) is independent from that giving Eqs. (32a) and (32b). In particular, as far as the Lorentz reciprocity theorem is concerned, a possible EM solution is a solution for which the couple of Eqs. (32a) and (33b), or Eqs. (32b) and (33a), applies. But the Lorentz reciprocity theorem does not tell us how the magneto-optic system has to be designed in order to exhibit this property.

According to the remark at the end of Section 2.1 Eqs. (32b), (33b) are valid without considering ${}^t[\varepsilon]$.

Notice that the elements of the $[r]$ and $[t]$ matrices involved in Eqs. (29a)–(29c), (32a), (32b) and (33a), (33b) are r and t-coefficients for the electric field.

Eqs. (32a) and (32b), (33a) and (33b) provide the desired relations between the off-diagonal elements of the transmission matrix: these elements are null (Eqs. (32a) and (33a)) or are linked by (32b) and (33b).

2.5. The second couple of solutions 1 and 2

The goal of this second choice is to derive relations bringing into play the diagonal elements of $[t]$. The two solutions are depicted in Fig. 3: solution 1 corresponds to an incident field in medium a whereas for solution 2 the incident field is in medium b. We consider successively, the two situations in which the two incident waves have the same polarization: p for solutions 1 and 2, s for solutions 1 and 2.

From Eq. (14a)–(14e) we get:

$$\vec{Q}^{a} = \vec{E}^{a}_{1,i} \times \vec{H}^{a}_{2,t} - \vec{E}^{a}_{2,t} \times \vec{H}^{a}_{1,i}, \tag{34a}$$

$$\vec{Q}^{b} = \vec{E}^{b}_{1,t} \times \vec{H}^{b}_{2,i} - \vec{E}^{b}_{2,i} \times \vec{H}^{b}_{1,t}. \tag{34b}$$

2.5.1. p-Polarized incident fields

Eq. (34a) yields:

$$\vec{Q}^{a} = \vec{E}^{a,p}_{1,i} \times \vec{H}^{a}_{2,t} - \vec{E}^{a}_{2,t} \times \vec{H}^{a,p}_{1,i}. \tag{35}$$

Use of Eqs. (16), (18a), (18b) and (20b) with the substitution $r \to t$ leads to:

$$\vec{Q}^{a} \cdot \vec{n}_y = \frac{2\omega\mu_0}{k_a^2}[H^{a,p}_{1,i,z}H^{a,p}_{2,t,z}](\vec{k}_{a,i} \cdot \vec{n}_y), \tag{36a}$$

$$\vec{Q}^{b} \cdot \vec{n}_y = -\frac{2\omega\mu_0}{k_b^2}[H^{b,p}_{2,i,z}H^{b,p}_{1,t,z}]\left(\vec{k}_{b,i} \cdot \vec{n}_y\right). \tag{36b}$$

Eqs. (11b) and (36a), (36b) give:

$$\frac{1}{k_a^2}[H^{a,p}_{1,i,z}H^{a,p}_{2,t,z}](\vec{k}_{a,i} \cdot \vec{n}_y) + \frac{1}{k_b^2}[H^{b,p}_{2,i,z}H^{b,p}_{1,t,z}](\vec{k}_{b,i} \cdot \vec{n}_y) = 0. \tag{37}$$

Defining new transmission coefficients by

$$H^{a,p}_{2,t,z}(y=0) = t^{pp}_{ba}(^{t}[\varepsilon])H^{b,p}_{2,i,z}(y=-e), \tag{38a}$$

$$H^{b,p}_{1,t,z}(y=-e) = t^{pp}_{ab}([\varepsilon])H^{a,p}_{1,i,z}(y=0), \tag{38b}$$

Eqs. (37) and (38a), (38b) give:

$$\frac{1}{k_a}t^{pp}_{ba}(^{t}[\varepsilon])\cos\theta_a = \frac{1}{k_b}t^{pp}_{ab}([\varepsilon])\cos\theta_b. \tag{39}$$

Contrary to what happens in Eqs. (32a), (32b) and (33a), (33b), the t-elements in Eq. (39) are transmission coefficients for the magnetic field.

2.5.2. s-Polarized incident fields

Making the transformation $\vec{E} \leftrightarrow \vec{H}$, $\varepsilon_0[\varepsilon] \leftrightarrow -\mu_0$, which leaves Maxwell equations invariant, Eqs. (36a) and (36b) becomes (together with $s \to p$):

$$\vec{Q}^{a} \cdot \vec{n}_y = \frac{-2}{\omega\mu_0}[E^{a,s}_{1,i,z}E^{a,s}_{2,t,z}](\vec{k}_{a,i} \cdot \vec{n}_y), \tag{40a}$$

$$\vec{Q}^{b} \cdot \vec{n}_y = \frac{2}{\omega\mu_0}[E^{b,s}_{2,i,z}E^{b,s}_{1,t,z}](\vec{k}_{b,i}.\vec{n}_y). \tag{40b}$$

We can again define new transmission coefficients by:

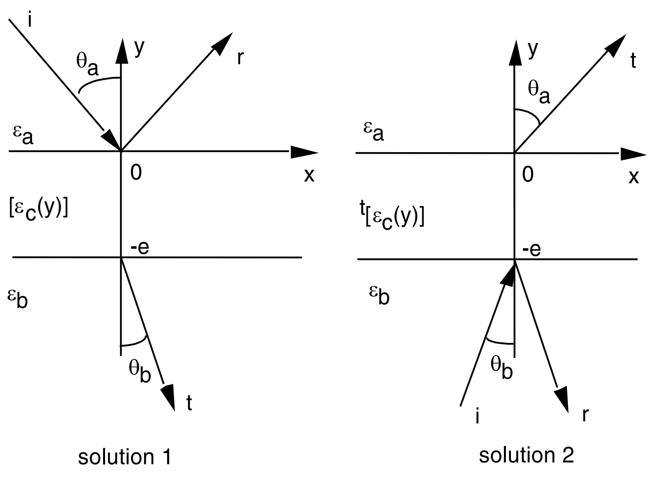

Fig. 3. Solutions 1 and 2 yielding the properties of the diagonal elements of $[t]$.

$$E^{a,s}_{2,t,z}(y=0) = t^{ss}_{ba}({}^{t}[\varepsilon])E^{b,s}_{2,i,z}(y=-e), \quad (41a)$$

$$E^{b,s}_{1,t,z}(y=-e) = t^{ss}_{ab}([\varepsilon])E^{a,s}_{1,i,z}(y=0). \quad (41b)$$

Use of Eqs. (11b), (40a), (40b), (41a) and (41b) leads to:

$$t^{ss}_{ba}({}^{t}[\varepsilon])k_a \cos\theta_a = t^{ss}_{ab}([\varepsilon])k_b \cos\theta_b. \quad (42)$$

The t-elements in Eq. (42) are transmission coefficients for the electric field.

From the remark at the end of Section 2.1, it is seen that Eqs. (39) and (42) can be rewritten as:

$$\frac{1}{k_a} t^{pp}_{ba}([\varepsilon])\cos\theta_a = -\frac{1}{k_b} t^{pp}_{ab}([\varepsilon])\cos\theta_b, \quad (43a)$$

$$t^{ss}_{ba}([\varepsilon])k_a \cos\theta_a = -t^{ss}_{ab}([\varepsilon])k_b \cos\theta_b. \quad (43b)$$

Eqs. (43a) and (43b) give the relation between the diagonal elements of the transmission matrix for p- and s-polarizations, respectively.

3. Numerical results

The magneto-optic system of interest is represented in Fig. 4. Using a computer code which makes a rigorous EM analysis of the device in Fig. 4, we have obtained the numerical values of the elements of the [r]- and [t]-matrices for polar and longitudinal orientations. This allows checking Eqs. (29b), (32b), (33b), (39) and (42). These values are reported in Appendix B: it is seen that the agreement is excellent since it is better than 10^{-13}.

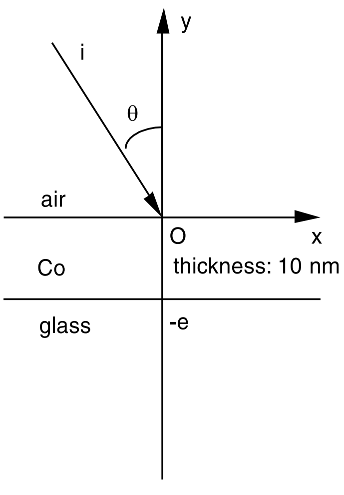

Fig. 4. The magneto-optic system used for the numerical calculations. Wavelength: 647 nm, $\theta_a = 30°$, θ_b is derived from Eq. (8), $e = 10$ nm; $[\varepsilon_{Co}] : \{\varepsilon_d = -11.5 + 18.3i,\ \varepsilon_{nd} = 0.65i\}$, $\varepsilon_b = 2.25$.

A careful examination of the numerical values reported in Eqs. (B.4a), (B.4b), (B.9a), (B.9b) on the one hand and Eqs. (B.5a) and (B.5b), (B.10a) and (B.10b) on the other hand show that the diagonal elements of the [t]-matrix very slightly depend on the magneto-optical orientation (of the order of 10^{-4}). This is quite natural when we look at the rigorous propagation Eq. (5a), while it is surprising when Eq. (5d) is considered. Indeed this slight dependence gives an order of magnitude of the error introduced by the approximation stated in Eq. (5c). This error is negligible as far as the EM field map along y is unchanged by the introduction of the extra-diagonal terms ε_{nd} created by the static magnetic field. This requires that the magneto-optical effect induces a variation of the index of refraction which remains small compared to the index steps at $y = 0, -e$.

4. Conclusion

In this paper, we have derived the relations fulfilled by the elements of the [r]- and [t]-matrices. For the sake of convenience, we summarize these relations:

$$|r^{ps}({}^{t}[\varepsilon])| = |r^{sp}([\varepsilon])|, \quad (44a)$$

$$|t^{ps}_{ba}({}^{t}[\varepsilon])|k_a \cos\theta_a = |t^{sp}_{ab}([\varepsilon])|k_b \cos\theta_b, \quad (44b)$$

$$|t^{sp}_{ba}([\varepsilon])|k_a \cos\theta_a = |t^{ps}_{ab}({}^{t}[\varepsilon])|k_b \cos\theta_b, \quad (44c)$$

$$\frac{1}{k_a} t^{pp}_{ba}({}^{t}[\varepsilon])\cos\theta_a = \frac{1}{k_b} t^{pp}_{ab}([\varepsilon])\cos\theta_b, \quad (44d)$$

$$t^{ss}_{ba}({}^{t}[\varepsilon])k_a \cos\theta_a = t^{ss}_{ab}([\varepsilon])k_b \cos\theta_b. \quad (44e)$$

Eqs. (44a)–(44e) are general in the sense that they apply to any planar multilayer and/or gradient permittivity magneto-optic device. Concerning the off-diagonal terms, we have found that they are either null or linked through their modulus: the relation between the off-diagonal coefficients depend on the structure at hand but whatever the system may be Eqs. (29a)–(29c), (32a), (32b) and (33a), (33b) are fulfilled. The Lorentz reciprocity theorem does not allow to get more specific relations. In other words, this theorem yields possible relations but does not tell us which magneto-optic device and which magneto-optical orientation will yield a given type of relation between the off-diagonal terms. An immediate application of such relations is that they allow to check computer codes developed in this domain. Another interest concerns the physics of this field since the results obtained in this paper point to general properties of magneto-optic systems.

Appendix A. Demonstration of Eq. (5a)

For the sake of convenience we rewrite Maxwell in medium c:

$$\mathrm{curl}\ \vec{E} = +\mathrm{i}\omega\mu_0\vec{H}, \tag{A.1a}$$

$$\mathrm{curl}\ \vec{H} = -\mathrm{i}\omega\varepsilon_0[\varepsilon_\mathrm{c}]\vec{E}. \tag{A.1b}$$

Eqs. (A.1a) and (A.1b) give:

$$\mathrm{curl}\,\mathrm{curl}\ \vec{E} = k_0^2[\varepsilon_\mathrm{c}]\vec{E} \tag{A.2a}$$

with

$$k_0 = \frac{\omega}{c}. \tag{A.2b}$$

Taking into account that

$$[\varepsilon_\mathrm{c}(y)] = [\varepsilon_\mathrm{d}(y)] + [\varepsilon_\mathrm{nd}(y)], \tag{A.3a}$$

Eq. (A.2a) can be rewritten as

$$\Delta\vec{E} + k_0^2\varepsilon_\mathrm{d}(y)\vec{E} = +\mathrm{grad}\,\mathrm{div}\ \vec{E} - k_0^2[\varepsilon_\mathrm{nd}(y)]\vec{E}, \tag{A.3b}$$

using the fact that $[\varepsilon_\mathrm{d}(y)]$ is diagonal with all its elements equal.

Besides Eq. (A.1b) shows that

$$\mathrm{div}[\varepsilon_\mathrm{c}(y)]\vec{E} = 0. \tag{A.4a}$$

But

$$\mathrm{div}[\varepsilon_\mathrm{c}(y)]\vec{E} = \varepsilon_\mathrm{d}(y)\mathrm{div}\vec{E} + \vec{E}\cdot\mathrm{grad}\varepsilon_\mathrm{d}(y) + \mathrm{div}[\varepsilon_\mathrm{nd}(y)]\vec{E}. \tag{A.4b}$$

Use of Eqs. (A.4a) and (A.4b) shows that Eq. (A.3b) writes

$$\Delta\vec{E} + k_0^2\varepsilon_\mathrm{d}(y)\vec{E} + \mathrm{grad}\left(E_y\frac{1}{\varepsilon_\mathrm{d}(y)}\frac{\mathrm{d}\varepsilon_\mathrm{d}}{\mathrm{d}y}\right) = -\mathrm{grad}\left[\frac{1}{\varepsilon_d(y)}\mathrm{div}[\varepsilon_\mathrm{nd}(y)]\vec{E}\right] - k_0^2[\varepsilon_\mathrm{nd}(y)]\vec{E}. \tag{A.5}$$

Appendix B. Check of the relations between the elements of the [*r*] and [*t*] matrices for polar and longitudinal magneto-optical orientations

The system of interest is represented Fig. 4.

B.1. Polar magneto-optical orientation

(a) off-diagonal reflection coefficient at $y = 0$:

$$r^\mathrm{sp}([\varepsilon]) = 0.0048956690932201 - \mathrm{i}0.0040829115856460, \tag{B.1a}$$

$$r^\mathrm{ps}({}^\mathrm{t}[\varepsilon]) = -0.0048956690932201 + \mathrm{i}0.0040829115856460. \tag{B.1b}$$

(b) off-diagonal transmission coefficients:

$$t_\mathrm{ba}^\mathrm{ps}({}^\mathrm{t}[\varepsilon])k_\mathrm{a}\cos\theta_\mathrm{a} = 0.0773827045319791 - \mathrm{i}0.0418363068465491\ \mu\mathrm{m}^{-1}, \tag{B.2a}$$

$$t_\mathrm{ab}^\mathrm{sp}([\varepsilon])k_\mathrm{b}\cos\theta_\mathrm{b} = 0.0773827045319792 - \mathrm{i}0.0418363068465492\ \mu\mathrm{m}^{-1}, \tag{B.2b}$$

$$t_\mathrm{ba}^\mathrm{sp}([\varepsilon])k_\mathrm{a}\cos\theta_\mathrm{a} = 0.0708921304202896 - \mathrm{i}0.0388047181599400\ \mu\mathrm{m}^{-1}, \tag{B.3a}$$

$$t_{ab}^{ps}({}^{t}[\varepsilon])k_b\cos\theta_b = 0.0708921304202905 - i0.0388047181599403\ \mu m^{-1}. \quad (B.3b)$$

(c) diagonal transmission coefficients:

$$\frac{1}{k_a}t_{ba}^{pp}({}^{t}[\varepsilon])\cos\theta_a = 0.0426595551666800 - i0.0059543524262004\ \mu m, \quad (B.4a)$$

$$\frac{1}{k_b}t_{ab}^{pp}([\varepsilon])\cos\theta_b = 0.0426595551666800 - i0.0059543524262004\ \mu m, \quad (B.4b)$$

$$t_{ba}^{ss}({}^{t}[\varepsilon])k_a\cos\theta_a = 5.4166590488384081 - i1.0059793811370530\ \mu m^{-1}, \quad (B.5a)$$

$$t_{ab}^{ss}([\varepsilon])k_b\cos\theta_b = 5.4166590488384081 - i1.0059793811370530\ \mu m^{-1}. \quad (B.5b)$$

B.2. Longitudinal magneto-optical orientation

(a) off-diagonal reflection coefficients at $y = 0$:

$$r^{sp}([\varepsilon]) = -0.0003084950261933 - i0.0002141972676732, \quad (B.6a)$$

$$r^{ps}({}^{t}[\varepsilon]) = -0.0003084950261932 - i0.0002141972676733. \quad (B.6b)$$

(b) off-diagonal transmission coefficients:

$$t_{ba}^{ps}({}^{t}[\varepsilon])k_a\cos\theta_a = -0.003440257989651 - i0.0037205984007549\ \mu m^{-1}, \quad (B.7a)$$

$$t_{ab}^{sp}([\varepsilon])k_b\cos\theta_b = -0.0034402527989649 - i0.0037205984007539\ \mu m^{-1}, \quad (B.7b)$$

$$t_{ba}^{sp}([\varepsilon])k_a\cos\theta_a = -0.0025529349717366 - i0.0029115821280467\ \mu m^{-1}, \quad (B.8a)$$

$$t_{ab}^{ps}({}^{t}[\varepsilon])k_b\cos\theta_b = 0.0025529349717376 - i0.0029115821280478\ \mu m^{-1}. \quad (B.8b)$$

(c) diagonal transmission coefficients:

$$\frac{1}{k_a}t_{ba}^{pp}({}^{t}[\varepsilon])\cos\theta_a = 0.0426660484068324 - i0.0059604484994219\ \mu m, \quad (B.9a)$$

$$\frac{1}{k_b}t_{ab}^{pp}([\varepsilon])\cos\theta_b = 0.0426660484068324 - i0.0059604484994219\ \mu m, \quad (B.9b)$$

$$t_{ba}^{ss}({}^{t}[\varepsilon])k_a\cos\theta_a = 5.4162931963705452 - i1.0045393835615539\ \mu m^{-1}, \quad (B.10a)$$

$$t_{ab}^{ss}([\varepsilon])k_b\cos\theta_b = 5.4162931963705461 - i1.0045393835615533\ \mu m^{-1}. \quad (B.10b)$$

The numerical values in Eqs. (B.1a), (B.1b) and (B.6a), (B.6b) are a check of Eq. (29b).

Concerning the coefficients of the transmission matrix, it is seen that the agreement is excellent between:

- the numerical values in Eqs. (B.2a), (B.2b), (B.7a), (B.7b) and Eq. (32b),
- the numerical values in Eqs. (B.3a), (B.3b), (B.8a), (B.8b) and Eq. (33b),
- the numerical values in Eqs. (B.4a), (B.4b), (B.9a), (B.9b) and Eq. (39),
- the numerical values in Eqs. (B.5a), (B.5b), (B.10a), (B.10b) and Eq. (42).

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