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Localization of light by a set of parallel cylinders

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Abstract. Using a rigorous theory of scattering, numerical evidence of the existence of localized modes in a one-dimensional or a two-dimensional set of circular dielectric rods is given. Particular attention is paid to the transition observed between the phenomena of propagation in periodic structures and localization phenomena in random structures. In particular, we show the strong connection between the phenomenon of a photonic band gap that appears in two-dimensional periodic sets of rods and the phenomenon of Anderson localization which is observed when the periodicity is broken

1. Introduction

The phenomenon of Anderson localization, discovered in the context of electron physics [1], has been investigated in the framework of classical fields like electromagnetic or acoustic fields as well (see for instance [2–7]). Recently, it has been shown from numerical results that localized modes can exist on randomly rough surfaces [8]. These modes, the so-called 'localitons', cannot propagate outside a small part of the surface. They decrease exponentially in time, due to radiation and absorption phenomena, but the shape of the mode remains fixed.

The first aim of our paper is to extend the notion of localiton to other scattering structures: a one-dimensional and a two-dimensional set of parallel dielectric rods. The second aim of the study is to describe the transition between the properties of a periodic set of rods and the properties observed when this periodicity is broken by a small random perturbation. It is shown that the non-localized propagating modes of a periodic set of rods become localitons due to the perturbation. In particular, we observe the link between a propagating mode located at the edge of a photonic band gap and a localized mode in the perturbed photonic crystal (the reader interested in the field of photonic crystals can find a review in [9]).

The computations have been made using a computer code elaborated from the rigorous theory described in [10]. This theory makes use of the notion of scattering matrix and the properties of translation of Bessel functions.

2. Propagation and localization of electromagnetic waves by dressed circular dielectric rods

2.1. *Propagation modes in the periodic set*

We consider in figure 1 a truncated periodic set of N circular dielectric rods ($N = 50$), illuminated by a wire S perpendicular to the figure. The rods and the wire

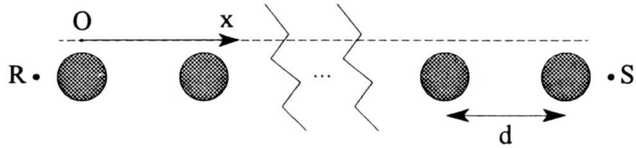


Figure 1. Truncated periodic set of N circular dielectric rods of index $\nu = 4$, diameter $D = 0{\cdot}402$, and period $d = 1$. This set is excited by a wire S located on the plane of symmetry of the rods, at a distance 0·21 from the centre of the first rod. We observed the total field at a point R located at the same distance from the centre of the last rod.

are infinite along the direction perpendicular to the figure. In such conditions, the field is s-polarized, and we draw in figure 2 (*a*) the energy received at point R located on the order side of the rods. More precisely, the ordinate in figure 2 (*a*) is the attenuation A defined by

$$A = \log_{10} \frac{|E_R|^2}{|\tilde{E}_R|^2}, \tag{1}$$

where E_R is the field received at point R and $\tilde{E}_R$ the field that would exist at point R after removing the rods. The most interesting feature of figure 2 (*a*) is the existence of discrete propagating modes. Another interesting feature of figure 2 (*a*) is the rapid variation of the attenuation from -5 to $+2$ at a wavelength close to 4·4. Figure 2 (*b*) shows that the phenomenon is not a discontinuity but appears progressively as the number of rods is increased. The enhancement of the field at point R for wavelengths greater than 4·4 is a surprising phenomenon we have not yet explained.

2.2. *Localized modes*

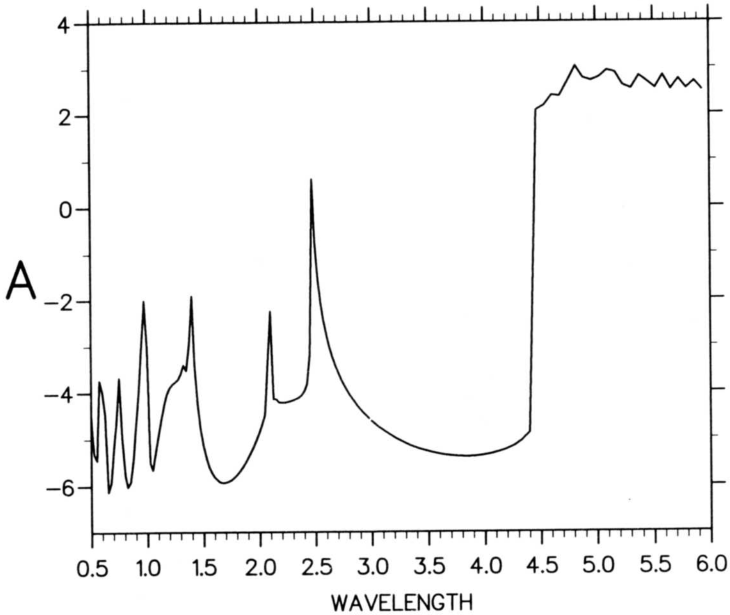
The structure of figure 1, with $N = 140$, has been perturbed by a uniform random translation δ of each rod along the x-axis:

$$-0{\cdot}12 \leqslant \delta \leqslant 0{\cdot}12. \tag{2}$$

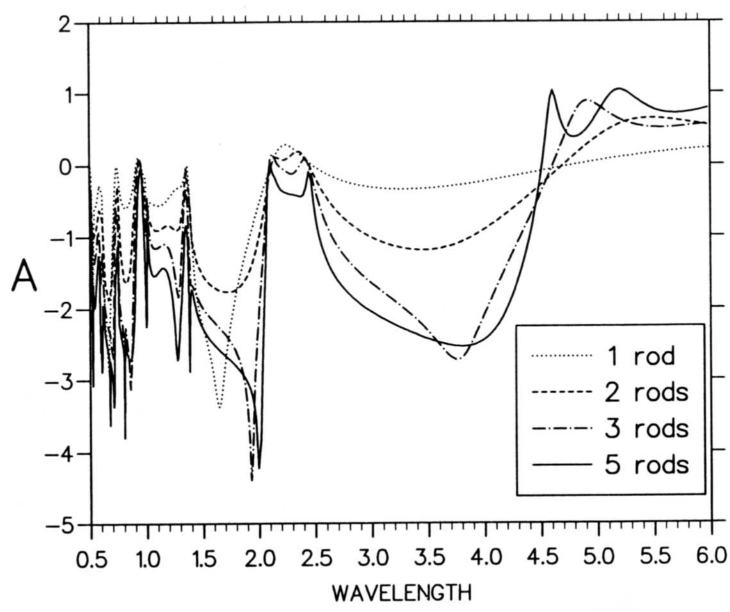
We have searched for homogeneous solutions of the scattering problem, viz. solutions existing without any source. Of course, these kinds of solutions of Maxwell's equations must decrease in time, due to the losses by radiation. In other words, the complex amplitude of this solution can be written in the form

$$E(x, y, t) = a(x, y) \exp(-\mathrm{i}\omega t), \tag{3}$$

with ω complex with a negative imaginary part, and $a(x, y)$ satisfying an outgoing wave condition in any direction. From a numerical point of view, the frequency ω is found by searching for the pole of the determinant of the scattering matrix associated with the set of rods in the complex lower plane of frequencies, using a classical Newton method. It is worth noticing that a complex value of ω leads to a complex value of the wavelength λ since $\lambda = 2\pi c/\omega$. Of course, if the imaginary part of ω is negative, that of λ is positive. This iteration method needs a prior estimate of the location of the pole. This pole has been searched for in the vicinity of the wavelength of 2·06, one of the transmission peaks of figure 2. The resonant wavelength has been found to be equal to $\lambda = 2{\cdot}064 + \mathrm{i}0{\cdot}005$. Figure 3 shows the modulus of the electric field as a function of x on a plane located at the top of the rods (see dashed line on figure 1). The most striking feature of this figure is the localization of the field between $x = 35$ and $x = 55$, i.e. over a range of about 20 rods.



(*a*)



(*b*)

Figure 2. (*a*) Attenuation by the set of rods of figure 1. (*b*) Same as figure 2 (*a*), for different values of N.

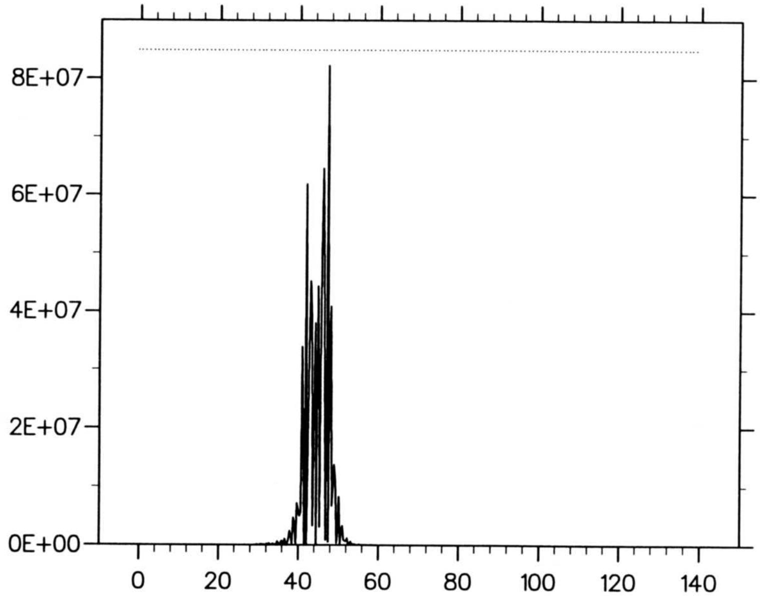


Figure 3. Square modulus of the localized mode of the structure of figure 1 after perturbation. The field has been computed on the plane represented by the dashed line of figure 1. The distance from this plane to the centres of the rods is equal to 0·21. The points at the top of the figure show the centres of the rods.

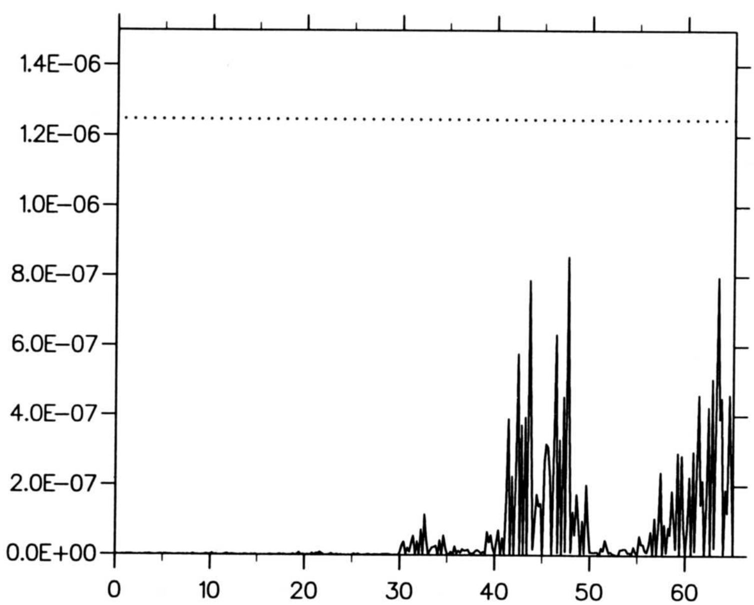


Figure 4. Excitation of a localiton by a propagating field.

This is obviously a generalization of the notion of localiton introduced in [8] in the framework of scattering by randomly rough surfaces. It is fundamental to notice that this localization phenomenon is a pure consequence of Maxwell equations and disorder. In other words, this phenomenon is observed numerically, but no condition of localization has been introduced in the theory.

2.3. *Excitation of localitons by propagating modes*

In order to exhibit the excitation of the localiton by a propagating mode, we have truncated the set of rods of figure 3 near the support to the localiton by keeping the 65 rods located on the left-hand side. Figure 4 shows the square modulus of the field at the top of the rods when the light source is a wire placed close to the right-hand side of the set of rods. The real wavelength of 2·064 corresponds to the real part of complex frequency of the localiton of figure 3. Obviously, the propagation wave has excited a resonance phenomenon on the support of the localiton.

3. Propagation and localization by a photonic crystal

3.1. *Propagation in a photonic crystal*

Figure 5 shows a photonic crystal formed by a periodic set of circular dielectric rods with hexagonal symmetry. This crystal is formed by the superposition of L horizontal grids containing, alternatively, N and N-1 circular rods. Figure 6 shows the transmittance of this crystal, for $N=\infty$ and $L=18$, illuminated by an s-polarized plane wave with normal incidence. This curve has been obtained using the code RESEAU 2000 [11], elaborated from a rigorous theory of gratings. The most interesting feature of this figure is the existence of a large transmission gap between $\lambda = 1{\cdot}85$ and $\lambda = 2{\cdot}6$, other small gaps being present for smaller wavelengths.

In order to study the structure of the field inside the crystal, we have used the code described in [10] and used in section 2. This structure is shown in figure 7, for various wavelengths. The wavelength of 1·7 is situated outside the gap, at the left side. Obviously, the field penetrates inside the crystal. At $\lambda = 1{\cdot}9$, situated at the left side extremity of the gap, the field is significant on the illuminated sides of the crystal but it rapidly vanishes inside. At a wavelength of 2·2 situated at the centre of the gap, the attenuation is much greater.

3.2. *Localization by a perturbed photonic crystal*

It is well known that at wavelengths placed inside the gap, small random perturbations are able to generate localization phenomena inside the crystal. Here, we have introduced in the crystal studied in the last paragraph a random displacement of the rods, with uniform distribution in the orthogonal x and y directions of the cross-section plane, with

$$-0{\cdot}18 \leqslant \delta_x \leqslant 0{\cdot}18 \tag{4}$$

$$-0{\cdot}18 \leqslant \delta_y \leqslant 0{\cdot}18. \tag{5}$$

Figure 8 shows the structure of two different localitons of the same structure. The real part of the first wavelength (1·9026 + i0·0028) is located at the left-hand side of the gap, while the real part of the second wavelength (2·34583 + i0·000866) is close to the centre of the gap. As a consequence, the fact that the imaginary part of the first wavelength is three times greater than that of the second one is very intuitive: when the wavelength is close to one extremity of the gap, the effect of the perturbation is stronger. This remark is illustrated in figure 9. It emerges that

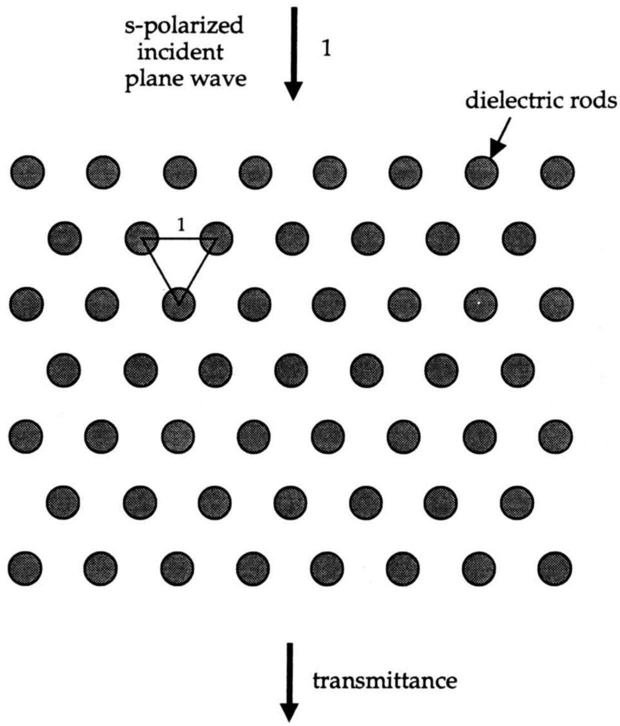


Figure 5. A photonic crystal of circular dielectric rods with hexagonal symmetry. Index $\nu = 2{\cdot}1$, diameter $D = 0{\cdot}42$, distance between neighbours $d = 1$. It is made by the superposition of L grids ($L = 7$ in the figure) having, alternatively, N and $N-1$ circular rods.

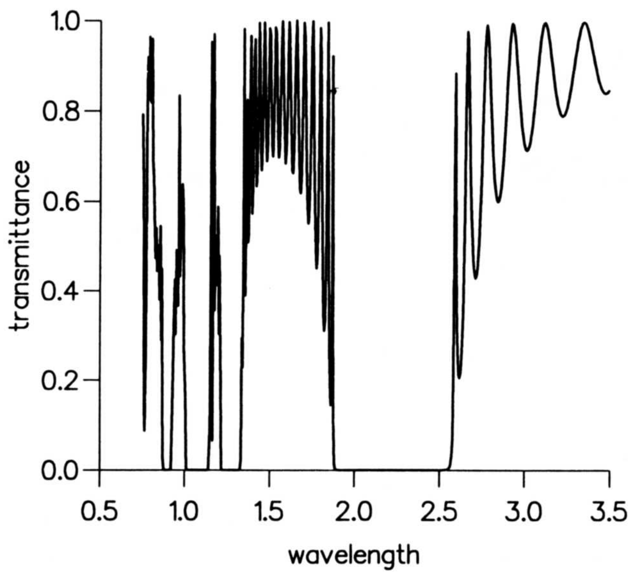


Figure 6. Transmittance of the photonic crystal of figure 5, with $N = \infty$, $L = 18$.

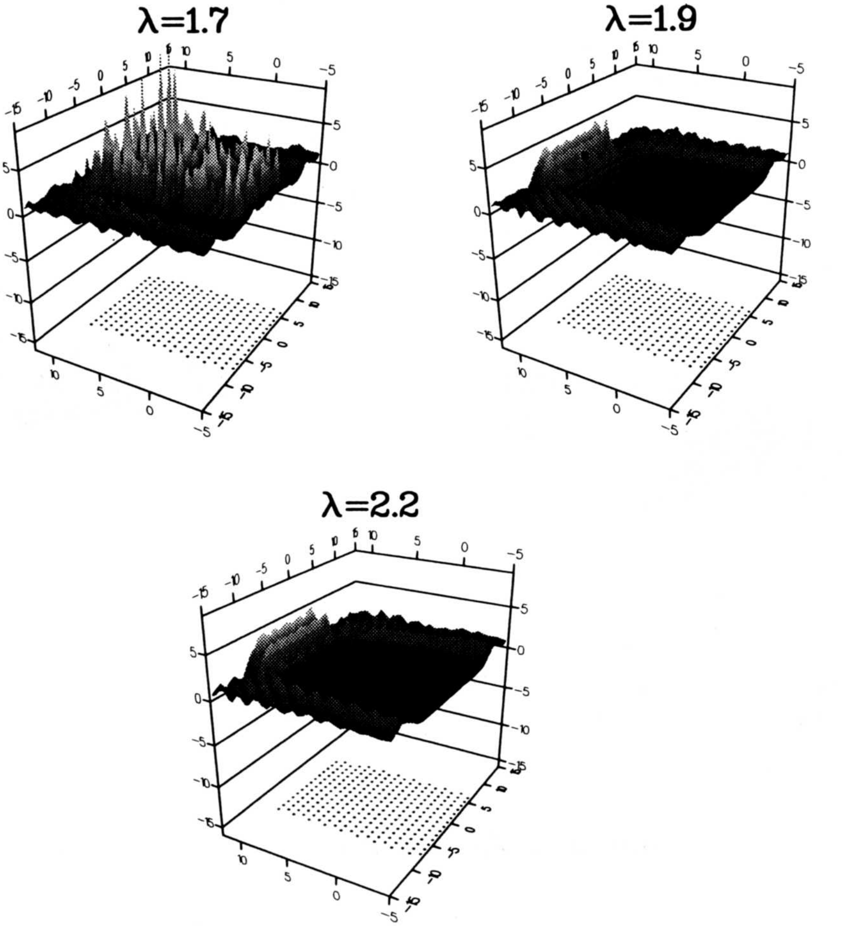


Figure 7. Square modulus of the field inside the photonic crystal of figure 5 with $N = 15$ and $L = 18$. The incident plane wave of unit amplitude comes from the left-hand side. The cross section of the structure is shown by the dots on the lower plane.

the width of the gap is reduced by the perturbation. The support of the localiton has a width of about 10, i.e., five wavelengths. A better demonstration of the existence of localized modes would require larger crystals, but computation time prevents us from achieving such computation.

3.3. *Excitation of localitons by an incident plane wave*

Figure 10 shows the field in the cross-section plane of the perturbed crystal when it is illuminated by a plane wave coming from the left-hand side of the figure for some real values of the wavelength. Obviously, as in figure 4, real wavelengths are able to excite a localiton provided the wavelength is close to the resonance wavelength of the localiton. Here, the wavelengths of 1·9 and 2·34583 excite the two localitons of figure 8. On the other hand, the maps of the field at $\lambda = 1{\cdot}7$ or $\lambda = 2{\cdot}2$ look similar to those of figure 7 (periodic crystal). The greatest attenuation of the field for $\lambda = 1{\cdot}7$ is clearly explained by figure 9: the perturbation entails a decrease of transmission on both sides of the gap.

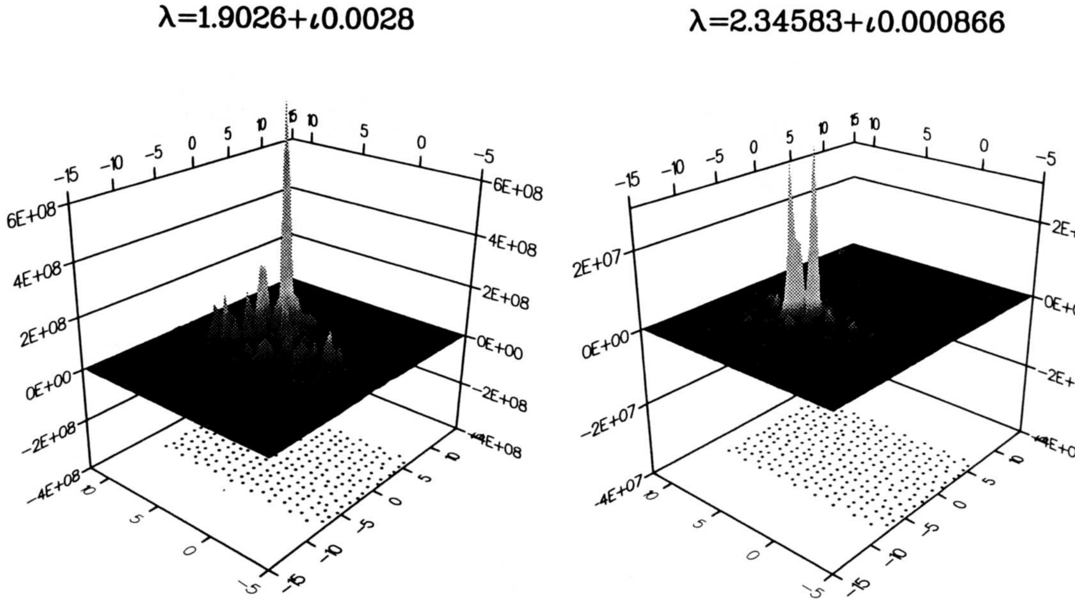


Figure 8. Square modulus of the localized modes of the structure of figure 5 after random perturbation. The points at the bottom of the figure show the centres of the rods.

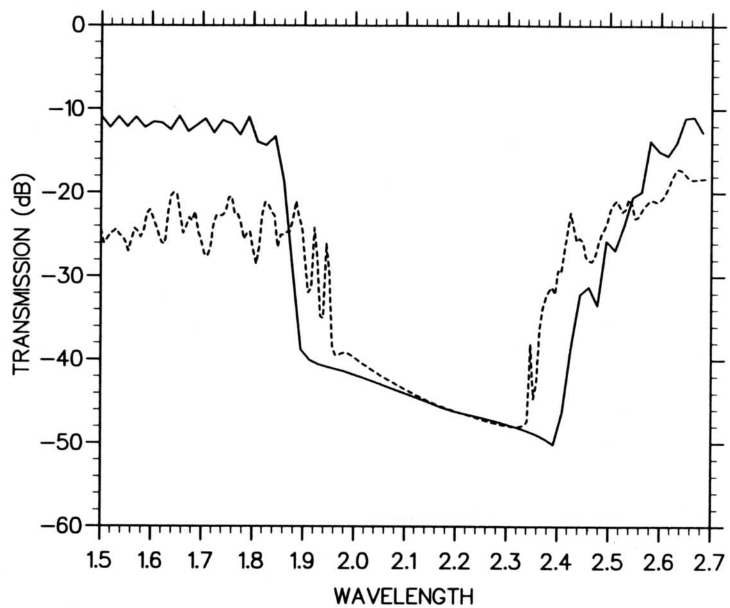


Figure 9. Flux of the Poynting vector on the face placed in the shadow region of the photonic crystal of figure 5 ($N = 15$, $L = 18$). Solid line: before the perturbation, dashed line: after perturbation.

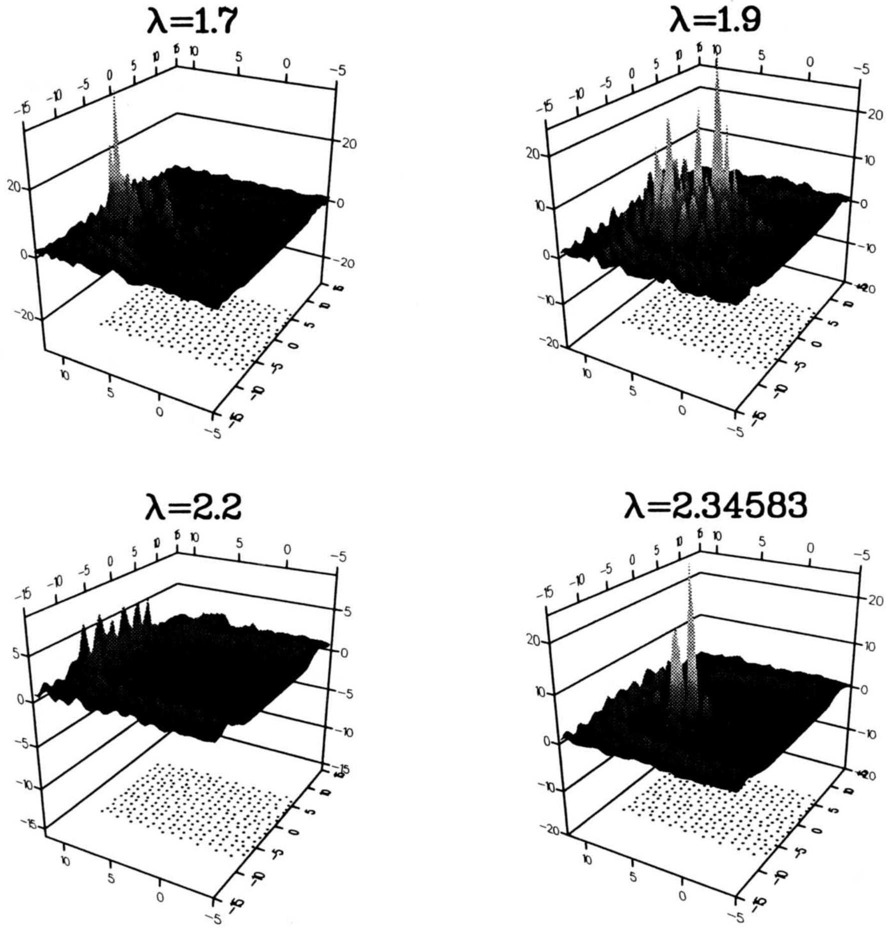


Figure 10. The same as figure 7, but with the perturbed crystal.

4. Conclusion

We have shown that the notion of localiton, defined for randomly rough surfaces, can be extended to many kinds of random structures, such as one-dimensional or two-dimensional arrays of dielectric rods.

It is noteworthy to notice that there exists a great difference between the localitons of these two structures. In the case of a one-dimensional structure, the imaginary part of the resonance wavelength of the localiton expresses the losses of energy by radiation on both sides of the array. This imaginary part should not decrease when the number of rods in increased. On the other hand, the imaginary part of the resonance wavelength of the localiton of a two-dimensional array is a direct consequence of the finite size of the crystal and should tend to zero when the side is increased. In other words, the localiton of an infinite crystal should have no loss. Indeed, it can be noticed that the imaginary parts of the resonance wavelengths of figure 3 are slightly greater than those of figure 8, even though the number of dressed rods is equal to 140 in figure 3, while the number of rods in one dimension of the crystal of figure 8 is of the order of 15 only. As a conclusion, it is fundamental to notice that for a two-dimensional set of rods, the notion of localiton, i.e. a homogeneous solution of Maxwell's equations in the complex plane of wavelengths, converges to the classical notion of localization. Indeed, the imaginary part of wavelength tends to zero as the size of the crystal is increased. On the other hand, for a one-dimensional set, the imaginary part of the wavelength is needed, in order to take into account the losses by radiation.

Acknowledgment

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