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Metamaterials: from microwaves to the visible region

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Abstract

In a recent paper, J.B. Pendry [J.B. Pendry, Negative refraction makes a perfect lens, Phys. Rev. Lett. 86 (2000) 3966–3969] has mentioned the possibility of making perfect lenses using a slab of left-handed material with relative permeability and permittivity equal to -1. He gave a demonstration of the vital influence of the evanescent waves in this process, arguing that these waves are amplified inside the slab. In the present paper, we first try to give a rigorous electromagnetic demonstration of Pendry's statement, and we show that in fact the integral expression of the field in a region of space diverges. Since this divergence does not prove that the perfect lens does not exist, we then give a very simple theoretical demonstration that a homogeneous material with both relative permittivity and permeability equal to -1 cannot exist, even for a unique frequency. However, thanks to the heterogeneous nature of a metamaterial, it is shown that a material able to focus light more efficiently than current devices (but not perfectly) could exist. Finally, it is shown that a plane slab of dielectric photonic crystal can also focus light, a property which could be crucial for construction of superlenses in the visible and infrared regions. ***To cite this article: D. Maystre et al., C. R. Physique 6 (2005).***

Résumé

Métamatériaux : des microondes au domaine visible. Dans un récent article, J.B. Pendry [J.B. Pendry, Negative refraction makes a perfect lens, Phys. Rev. Lett. 86 (2000) 3966–3969] a mentionné la possibilité d'élaborer des lentilles parfaites en utilisant une couche plane de matériau main gauche dont les permittivité et perméabilité relatives sont égales à -1. Dans sa démonstration, Pendry montre l'importance capitale des ondes évanescentes dans cette propriété, ces ondes subissant une amplification dans le matériau main gauche. Dans cet article, nous donnons d'abord une démonstration électromagnétique rigoureuse du résultat énoncé par Pendry. Nous relevons qu'en fait, l'expression intégrale du champ diverge dans une certaine région de l'espace. Cette remarque ne démontre évidemment pas que la lentille parfaite n'existe pas. Nous donnons donc une démontration théorique simple qu'un métamatériau possédant une permittivité et une perméabilité relatives égales à -1 ne peut exister, même à une seule fréquence. Toutefois, en prenant en compte la nature hétérogène d'un métamatériau, on peut montrer qu'un matériau capable de focaliser la lumière plus efficacement qu'un dispositif classique (mais non parfaitement) peut exister. Finalement, nous montrons qu'une couche planaire d'un cristal photonique diélectrique peut lui aussi focaliser la lumière, une propriété qui pourrait se révéler cruciale pour la construction de superlentilles dans les domaines visible et microonde. ***Pour citer cet article : D. Maystre et al., C. R. Physique 6 (2005).***

Keywords: Metamaterials; Left-handed materials; Negative refraction; Superlens; Electromagnetic theory; Photonic crystals

Mots-clés : Métamatériaux ; Matériaux main gauche ; Réfraction négative ; Superlentille ; Théorie électromagnétique ; Cristaux photoniques

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1. Introduction

Left-handed materials have raised a growing interest in the last years [1–17]. Veselago [13] in the 1960s, predicted surprising properties of negative refraction (with optical index equal to –1) for a material with both relative permittivity and permeability equal to -1. More recently, Pendry announced a remarkable property [1]: a slab of left-handed material constitutes a perfect, stigmatic lens. According to Pendry, the explanation of this phenomenon relies on the fact that, when the slab is illuminated by a point source, the evanescent part of the incident field, and not only the radiative part, contributes to the focusing process, allowing formation of a perfect image. It has been proposed to design left-handed materials in the microwave domain, using metal-dielectric periodic structures, such as periodic arrays of copper split ring resonators and wires [18–20]. Making such metal-dielectric structures at the optical scale seems very difficult. Furthermore, it should be remembered that metals in the optical region present losses. However, a recent paper [12] has reported the possibility of making materials presenting negative refraction phenomena using simple dielectric photonic crystals, which makes it possible to design such structures in the optical region.

In this paper, we first deal with the notion of perfect lens using the rigorous laws of Electromagnetics. We show that the rigorous electromagnetic demonstration of the property of perfect focusing is incorrect since the integral electromagnetic expression of the field diverges in a given region of space, a fact which, of course, does not entail that the result given by Pendry is false. In fact, a very simple theoretical demonstration based on the notion of the analytic continuation of the field allows us to show that a material having both relative permittivity and permeability equal to -1 cannot exist. Furthermore, we show that a strong limitation to the realization of perfect lenses lies actually on the heterogeneous nature of the material: the concept of effective medium cannot be used for a slab of left-handed metamaterial illuminated by a point source, due to the vital influence of evanescent waves having a transverse wavelength of the order or less than the dimensions of the dielectric and metallic inclusions.

Surprisingly, the consequence of this explanation is that a slab of left-handed material could provide a means to focus light more efficiently than current devices, such as classical optical lenses, provided the size of the elementary cell of the material is small enough.

Finally, we give a numerical demonstration that a very classical dielectric photonic crystal is able to focus light. This property could make it possible to construct superlenses in the visible and infrared regions, where metal-dielectric structures have important losses.

2. Negative refraction of propagating waves, amplification of evanescent waves, Pendry's perfect lens

Let us calculate the plane wave expansions of the field generated by a two-dimension (2D) point source (or in other words a line source) illuminating a slab of left-handed material in the three regions of space.

The point source S is located at a distance y_s above the upper interface of the slab of left-handed material having a relative permittivity ε and a relative permeability μ equal to -1 at frequency ω. The width of the slab is equal to e and throughout the paper, we assume that $e > y_s$. The field is s-polarized, with the electric field parallel to the z-axis of a Cartesian coordinates system xyz, the x-axis being located at the upper interface (Fig. 1).

Using a time dependency in $\exp(-\mathrm{i}\omega t)$, the field emitted by the point source S of ordinate y_S at a point M is given by

$$E^S = H_0^{(1)}(k_0 SM) \tag{1}$$

with $k_0 = 2\pi/\lambda_0$ the wavenumber in vacuum ($k_0 = \omega/c$, c velocity of light in vacuum and λ_0 wavelength in vacuum) and $H_0^{(1)}$ the Hankel function of the first kind and zeroth order. Let us recall that this function is nothing but the Green function of the Helmholtz equation in vacuum, which satisfies

$$\nabla^2 E^s + k_0^2 E^s = 4\mathrm{i}\delta(x)\delta(y - y_s) \tag{2}$$

with $\delta(x)\delta(y - y_s)$, the Dirac distribution, placed at the point source. Let us call $E(x, y)$ the total field generated at any point of space by this point source. Bearing in mind that $\varepsilon\mu = 1$ in the left-handed material, the field satisfies in the three regions of space the Helmholtz equation

$$\nabla^2 E + k_0^2 E = 4\mathrm{i}\delta(x)\delta(y - y_s) \quad \text{if } y > 0 \tag{3}$$

$$\nabla^2 E + k_0^2 E = 0 \quad \text{if } -e < y < 0 \tag{4}$$

$$\nabla^2 E + k_0^2 E = 0 \quad \text{if } y < -e \tag{5}$$

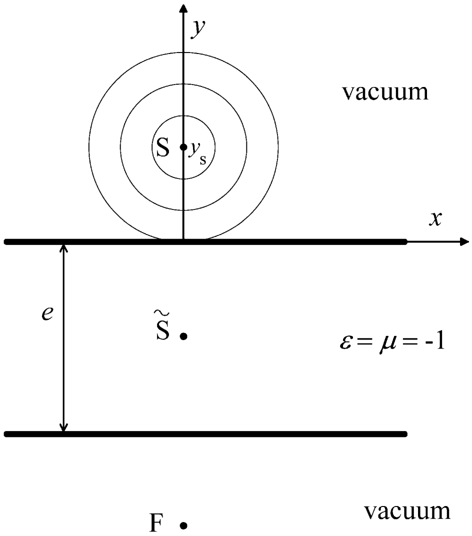

Fig. 1. Presentation of the problem.

Fig. 1. Présentation du problème.

At the interfaces between the different media, E and $\frac{1}{\mu}\frac{\partial E}{\partial y}$ are continuous, or in other words, the field is symmetric and the derivative in y is anti-symmetric on both interfaces. Using the Weyl formula [21], the Hankel function can be expressed at a point M in the form of a sum of plane waves above and below the point source:

$$H_0^{(1)}\{k_0 SM\} = \frac{1}{\pi} \int_{\alpha=-\infty}^{+\infty} \frac{1}{\beta(\alpha)} \exp\{\mathrm{i}\alpha x + \mathrm{i}\beta(\alpha)|y - y_S|\}\,\mathrm{d}\alpha \tag{6}$$

with

$$\beta(\alpha) = \sqrt{k_0^2 - \alpha^2} \text{ or } \mathrm{i}\sqrt{\alpha^2 - k_0^2} \tag{7}$$

Below the point source, $y - y_S$ is negative and Eq. (6) takes the form

$$H_0^{(1)}\{k_0 SM\} = \frac{1}{\pi} \int_{\alpha=-\infty}^{+\infty} \frac{1}{\beta(\alpha)} \exp\{\mathrm{i}\alpha x - \mathrm{i}\beta(\alpha)(y - y_S)\}\,\mathrm{d}\alpha = \frac{1}{\pi} \int_{\alpha=-\infty}^{+\infty} \frac{\exp\{\mathrm{i}\beta(\alpha) y_S\}}{\beta(\alpha)} \exp\{\mathrm{i}\alpha x - \mathrm{i}\beta(\alpha) y\}\,\mathrm{d}\alpha \tag{8}$$

This equation shows that the slab is illuminated by a sum of plane waves, including propagating and evanescent ones. In the present paper, we call evanescent a wave which propagates parallel to the slab and behaves exponentially along the orthogonal direction.

Let us calculate the field transmitted below the slab by each of these incident waves. With this aim, we consider the (possibly evanescent) incident wave propagating above the slab

$$E^{\mathrm{i}} = \exp\{\mathrm{i}\alpha x - \mathrm{i}\beta(\alpha) y\} \tag{9}$$

The total field generated above the slab by this plane wave can be written:

$$0 < y < y_S, \quad E = \exp\{\mathrm{i}\alpha x - \mathrm{i}\beta(\alpha) y\} + r \exp\{\mathrm{i}\alpha x + \mathrm{i}\beta(\alpha) y\} \tag{10}$$

Inside the slab, since ε and μ are equal to -1, the wavenumber, and thus the propagation constant $\beta(\alpha)$ remains unchanged and, taking into account the reflections on both sides of the slab,

$$-e < y < 0, \quad E = a \exp\{\mathrm{i}\alpha x - \mathrm{i}\beta(\alpha) y\} + b \exp\{\mathrm{i}\alpha x + \mathrm{i}\beta(\alpha) y\} \tag{11}$$

It is worth noting that we make no assumption on the shape of the field inside the slab: both possibilities (up-going and down-going y-propagating waves, y-increasing and y-decaying, termed as anti-evanescent and evanescent in the following) are included in our expression of the field. The same remark can be made on the calculations made in [15–17] which envisage successively all the possibilities of decaying and decreasing waves.

Finally, below the slab, the radiation condition in vacuum allows us to write the transmitted field in the form:

$$y < -e, \quad E = t \exp\{\mathrm{i}\alpha x - \mathrm{i}\beta(\alpha) y\} \tag{12}$$

In order to calculate the coefficients a, b, r, t, we must express the boundary conditions on both interfaces of the slab. From the continuity of E, it turns out that

$$1 + r = a + b \tag{13}$$

$$a \exp\{\mathrm{i}\beta(\alpha)e\} + b \exp\{-\mathrm{i}\beta(\alpha)e\} = t \exp\{\mathrm{i}\beta(\alpha)e\} \tag{14}$$

and from the continuity of $\frac{1}{\mu}\frac{\partial E}{\partial y}$

$$-1 + r = -(-a + b) = a - b \tag{15}$$

$$-a \exp\{\mathrm{i}\beta(\alpha)e\} + b \exp\{-\mathrm{i}\beta(\alpha)e\} = -\big(-t \exp\{\mathrm{i}\beta(\alpha)e\}\big) = t \exp\{\mathrm{i}\beta(\alpha)e\} \tag{16}$$

From Eqs. (13)–(16), it comes out from straightforward calculations

$$r = a = 0 \tag{17}$$

$$b = 1 \tag{18}$$

$$t = \exp\{-2\mathrm{i}\beta(\alpha)e\} \tag{19}$$

It is worth noticing that inside the slab, if $\beta(\alpha)$ is real, the phase velocity is oriented towards the upper interface. When $\beta(\alpha)$ is imaginary, the amplitude of the evanescent wave does not decay away exponentially from the upper interface but increases exponentially. These properties could seem unphysical, but it must be recalled, firstly, that the propagation of energy is linked to the group velocity (which in a left-handed material is just the opposite of the phase velocity) and, secondly, that an evanescent wave does not propagate energy in the y-direction. Anyway, in the present paper, we have not made any hypothesis about the behavior of the wave inside the slab and our result is a direct consequence of the elementary laws of electromagnetism. It is worth noticing that the model of a slab differs from that of a semi-infinite medium, which is used in the first part of [1].

Finally, from Eqs. (8), (12) and (19), it is possible to express the field generated inside and below the slab by the point source

$$-e < y < 0, \quad E = \frac{1}{\pi} \int_{\alpha=-\infty}^{+\infty} \frac{1}{\beta(\alpha)} \exp\{\mathrm{i}\alpha x + \mathrm{i}\beta(\alpha)(y + y_S)\}\,\mathrm{d}\alpha \tag{20}$$

$$\begin{aligned} y < -e, \quad E &= \frac{1}{\pi} \int_{\alpha=-\infty}^{+\infty} \frac{\exp\{\mathrm{i}\beta(\alpha) y_S\}}{\beta(\alpha)} \exp\{-2\mathrm{i}\beta(\alpha)e\} \exp\{\mathrm{i}\alpha x - \mathrm{i}\beta(\alpha) y\}\,\mathrm{d}\alpha \\ &= \frac{1}{\pi} \int_{\alpha=-\infty}^{+\infty} \frac{1}{\beta(\alpha)} \exp\{\mathrm{i}\alpha x - \mathrm{i}\beta(\alpha)(y - y_F)\}\,\mathrm{d}\alpha \end{aligned} \tag{21}$$

$$y_F = y_S - 2e \tag{22}$$

In the slab, at the ordinate $-y_S$ of the point $\tilde{S}$, it emerges from Eq. (20) that the field is identical to that obtained at the ordinate $+y_S$, which includes the point source S (see Eq. (8)). This property shows that $\tilde{S}$ is a point image. The same property holds at the ordinate $y_F = y_S - 2e$ of the point F.

From this section, it emerges that the slab of left-handed material has generated two stigmatic images of the point source, the first one in the slab, the second one below the slab. It is worth noting that the image located below the slab is deduced from the point source through a vertical translation of $-2e$. Thus, the image of an object located above the slab will be an identical object located below the slab.

3. Criticisms on Pendry's perfect lens

3.1. Divergence of the expression of the field between two images

In this section, it is shown that the expression of the field given in Section 2 diverges in a region of space. Inside the slab, below $\tilde{S}$, $y + y_S$ becomes negative and the integrand I in the right-hand member of Eq. (20) increases exponentially as $|\alpha|$ tends to infinity, a fact already shown in [16,17]. The same remark holds for the expression given by Eq. (21) of the field between the bottom of the slab and the point image F given by, since $y - y_F$ becomes positive.

Do these remarks authorize us to claim that the result about the perfect lens stated by Pendry fails? We do not think so. First, a correct result can be obtained from an incorrect demonstration. Secondly, a problem of divergence of integrals or series can sometimes be solved using a new mathematical definition of the convergence. For example, the Padé theory allows one to give a mathematical sense to series which obviously diverge from the point of view of the physicist.

Thus, in order to demonstrate that the perfect lens does not exist, it is necessary to show that it violates a basic principle of Physics. This is what we do in the next subsection.

3.2. *Why a left-handed material with $\varepsilon = \mu = -1$ does not exist*

Let us define a function $U(x, y)$ for $y > -e$ by

$$U(x, y) = \begin{cases} E(x, y) & \text{if } y > 0 \\ E(x, -y) & \text{if } -e < y < 0 \end{cases} \tag{23}$$

This function is symmetrical with respect to the x-axis, which entails that $\frac{\partial U}{\partial y}$ is anti-symmetrical, or in other words that the y-derivative of U identifies with the y-derivative of E on both sides of the x-axis. Since both E and U are continuous, it emerges that the function $F = E - U$ and its y-derivative are continuous on the x-axis. Finally, these functions satisfy in the two upper regions the Helmholtz equation in the sense of distributions, except maybe at the point source and at the symmetrical point $\tilde{S}$ with respect to the x-axis (due to the term $E(x, -y)$ in the definition of U in Eq. (23)). In fact, the right-hand member of Eq. (3) satisfied by E and located at the point source disappears for F since, by definition, F vanishes for $y > 0$. In conclusion, due to the its continuity and the continuity of its y-derivative, F satisfies for $y > -e$ the homogeneous Helmholtz equation, except maybe at the point $(0, -y_s)$ where a Dirac function could appear in the right-hand member.

The consequence of this property is crucial. Indeed, a mathematical theorem states that *a function satisfying the Helmholtz equation in a given domain and known in a given sub-domain can be analytically continued in a unique manner in the entire domain* [21–23]. It is important to notice that this theorem deals with the analytic continuation of functions of two variables satisfying a Helmholtz equation in the xy plane and not with the analytic continuation of functions of the complex variable $x + \mathrm{i}y$ in the complex plane. A straightforward consequence of this theorem is that if this function vanishes in a sub-domain, it vanishes in the entire domain. Here, F vanishes in the sub-domain $y > 0$, thus it vanishes in the entire domain where Helmholtz equation is satisfied, $y > -e$, including at the point $(0, -y_s)$ since obviously the continuation of F has no singularity at this point. Thus it turns out that $E(x, -y)$ and $E(x, y)$ identify when $-e < y < 0$. Since $E(x, -y)$ (field above the interface $y = 0$) has a singular part $E^S = H_0^{(1)}(k_0 SM)$ and since the field reflected by the interface $y = 0$ has no singularity above this interface, we deduce that if $e > y_s$, $E(x, y)$ has the same singularity below the interface $y = 0$ and contains a singular part

$$\tilde{E}^S = H_0^{(1)}(k_0 \tilde{S} M) \tag{24}$$

This behavior is not acceptable since the field transmitted inside the film cannot have singularities or, in other words, the field inside the film cannot contain a point-source, since we have assumed that the only point source was located above the slab. We are led to the conclusion that a left-handed material with both relative permittivity and permeability equal to -1 cannot exist, even at a unique wavelength, since it violates a fundamental principle of Electromagnetics: the field generated by a unique point source cannot include another point source.

Our conclusion on the impossibility of elaborating a homogeneous material having both relative permittivity and permeability equal to -1 seems to be in contradiction with experimental results [20] which showed the phenomenon of negative refraction in metallo-dielectric metamaterials. The aim of the next section is to show that this paradox can be explained by the limits of validity of the homogenization process for bounded objects made with metamaterials.

Furthermore, it will be shown that metallo-dielectric metamaterials could provide a means for constructing remarkable (but non-perfect) lenses.

3.3. *Why a material close to a left-handed material with $\varepsilon = \mu = -1$ can exist*

It has been shown that the evanescent and anti-evanescent waves play a vital role in the existence of a divergence of the field inside and below the slab. Let us introduce the transverse wavelength $\lambda_T = 2\pi/\alpha$ of such a wave, α being the propagation constant on the x-axis of this evanescent or anti-evanescent wave. This transverse wavelength is the period of the field on the x-axis, which is the direction of propagation of the evanescent waves. In experimental devices, the metamaterials have a periodic structure, with a period typically 6 times smaller than the wavelength of the light [20]. Let us denote by d the period of the metamaterial on the x-axis. As far as the transverse wavelength λ_T remains much greater than the period d, one can predict that the homogenization process, which assumes that the heterogeneous material can be replaced by an homogeneous material with effective relative permittivity and permeability (here equal to -1) is valid. On the other hand, when the order of magnitude of λ_T approaches or becomes smaller than d, obviously the homogenization process fails since the field presents variations at the scale of the period of the structure. This phenomenon is very well known in the domain of near-field microscopy. The use of optical antennas placed very close to a rough surface allow the specialists of this technique to generate incident fields containing evanescent waves with transverse wavelengths very small with respect to the actual wavelength of the light source. As a consequence, they are able to determine details of the roughness less than 10^{-2} or 10^{-3} wavelengths [24]. This is the proof that the phenomenon of scattering generated by such incident waves on a rough surface is strongly influenced by the asperities, even though they are much smaller than the wavelength. In our case, the period of the metamaterial plays the same role as the size of the asperities.

Furthermore, some specialists of the homogenization process have stressed the limits of the notion of effective medium for bounded objects. In contrast with most of the studies in this field, which assume that the heterogeneous material is unbounded and that the wavelength tends to infinity, they keep constant the bounded shape of the object and the wavelength, while the period of the heterogeneous material tends to 0 [25]. In our case, the interfaces of the slab allow propagation of evanescent and anti-evanescent waves along the x-axis and this is the origin of the inadequacy of the homogenization process. On the other hand, in a given range of values of α, the transverse wavelength is much larger than the period of the crystal and the homogenization process is valid. Let us show that this property allows us to predict that, even though the perfect lens cannot exist, slabs of left-handed materials could provide a means to elaborate lenses with very strong focusing properties, provided the period of the metamaterial is small enough.

From the studies on homogenization, it emerges that the approximate limit of validity of homogenization of a structure with period d in which a wave of period $\lambda_T = 2\pi/|\alpha|$ propagates is given by

$$\lambda_T > 2\pi d \tag{25}$$

which can be written

$$|\alpha| < \frac{1}{d} \tag{26}$$

This means that the part of the integrals of Eqs. (20) and (21) exterior to the interval given by Eq. (26) has no physical meaning, since outside this interval, the material cannot be considered as homogeneous. In other words, only the part of the integral located inside this range contributes to the phenomenon of focusing and in practice, the integral must be limited to this finite range, called homogenization region in Fig. 2.

Of course, all materials present heterogeneities, including those that can be found in nature. However, for these media, the size of the heterogeneities is of the order of a nanometer. It is not so in left-handed materials, where the heterogeneities have the same order of magnitude as the wavelength of light and consequently have a crucial effect on propagation properties.

The images inside and below the slab are no longer points. The order of magnitude of the size of the image, deduced from Eq. (26), is equal to $2\pi d = \lambda_T$. If λ_0/d has the same order of magnitude as in the first experiments (of the order of 6), the size of the image is equal to one wavelength. This is the size of an image provided by a perfect classical two-dimensional lens, the width of which is much greater than its distance to the point source. Better resolution could be obtained, provided that technological means allow one to elaborate left-handed materials with larger values of λ_0/d. In that case, a part of the evanescent part of the incident field will contribute to the image formation. Thus, the smaller the period, the better the resolution. It is interesting to compare the resolution of superlenses made of metamaterials with that of classical optical lenses. For classical lenses, evanescent

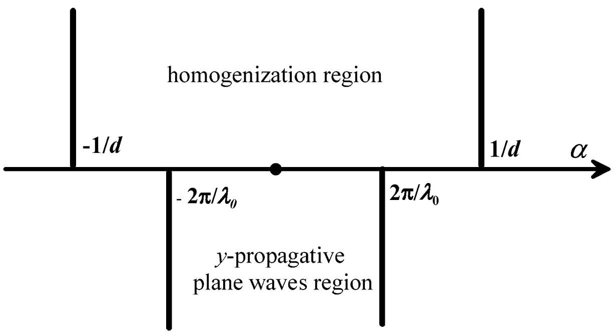

Fig. 2. The different regions for the wavenumber α on the x-axis.

Fig. 2. Les différents domaines du nombre d'onde α sur l'axe des x.

waves cannot be amplified, in such a way that the only waves which contribute to the focusing process are the propagative plane waves. From a mathematical point of view, this means that the best resolution is obtained by having a range of integration in Eq. (21) extending from $-k_0$ to $+k_0$, this case corresponding to an optical lens with an infinite diameter (y-propagative plane waves region in Fig. 2). Obviously, the superlens using left-handed materials should have a better resolution than classical lenses provided that the period of the metamaterial is small enough. An example of superlens with very high resolution is given in [26].

4. Metamaterials in the visible and infrared regions

It is well known that in the visible and near infrared regions, the metals are lossy. Thus, it can be conjectured that the losses of metallo-dielectric left-handed materials could cause problems in the focusing process. In this section, it is shown from numerical calculations that a dielectric photonic crystal can also be used to design a flat lens.

We consider a 2D dielectric photonic crystal (Fig. 3) made of 364 circular air galleries of diameter 0.588 inside a rectangular dielectric bulk of size 27.9 by 5.4 and relative permittivity $\varepsilon = 12$, forming a crystal of hexagonal symmetry, with distance $d = 0.68$ between two adjacent rods. It is illuminated by a line source (black cross in Fig. 3) parallel to the rods and located at a distance 2.7 above the top of the crystal. The wavelength is $\lambda_0 = 2.02$. The field map shows the existence of two images of the line source, the first one inside the crystal, symmetrical to the line source with respect to the top of the crystal, the second one below the crystal, at a distance from the line source equal to two times the width of the crystal. This result obeys the conclusion of Section 2, at least for the locations of the images. On the other hand, it emerges that the light intensities of the images are much smaller than that on the line source. In order to explain this result, it must be noticed that the phenomenon of negative refraction is not restricted to a material with relative permittivity and permeability close to -1. In fact, this phenomenon arises as soon as these parameters are negative, the optical index being equal to $-\sqrt{\varepsilon\mu}$. However, in that case, the transmission of energy on the boundaries of the crystal is no longer perfect, and the amplification of the evanescent waves disappears. It has been

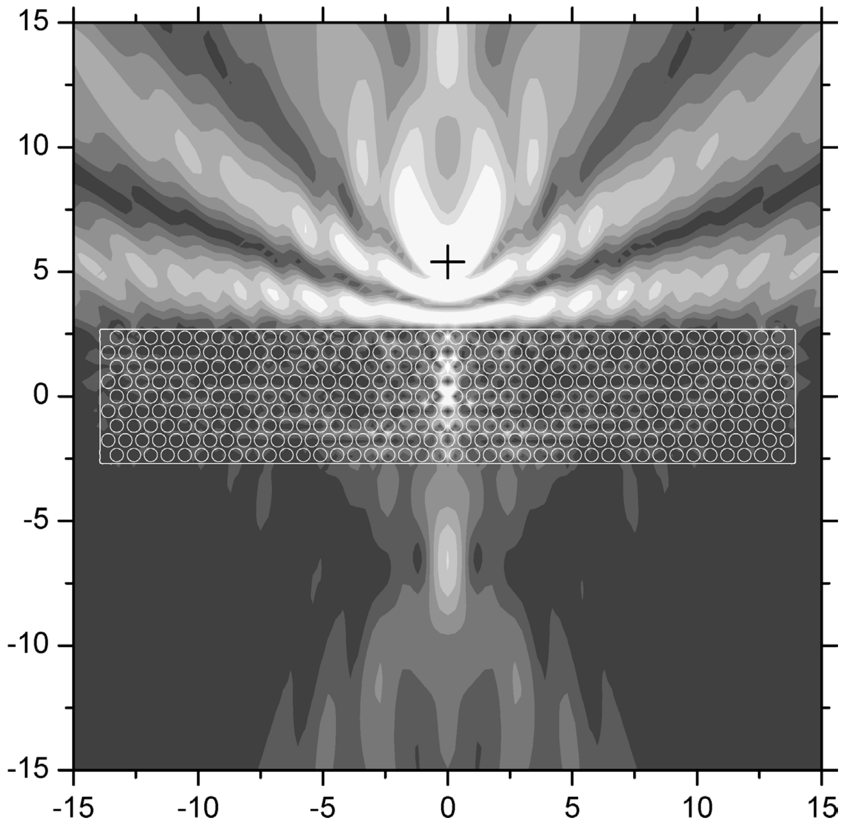

Fig. 3. Focusing of light by a dielectric photonic crystal. The map shows the modulus of the electric field.

Fig. 3. Focalisation de la lumière par un cristal photonique diélectrique. La carte donne le module du champ électrique.

verified that the effective index of the photonic crystal is equal to -1 and we deduce that $\sqrt{\varepsilon\mu} = 1$. In Fig. 3, the width of the image located below the bottom of the crystal is close to λ. Thus, this resolution corresponds exactly to the case where $\lambda_T = \lambda$, or in other words to the case where no evanescent wave is amplified, the focusing process being generated by the phenomenon of negative refraction for all the propagating waves. We actually try to determine the values of the effective permittivity and permeability.

5. Conclusion

In conclusion, a rigorous electromagnetic analysis of the field transmitted inside a slab of left-handed material illuminated by a monochromatic field has allowed us to prove that this slab cannot act as a perfect lens, since a material with both relative permittivity and permeability equal to -1 is able to produce point sources in the transmitted field and thus violates basic principles of electromagnetism. It must be noticed that this result was obtained from two approaches, using respectively the analytic continuation of the field inside the slab and the behavior of the integral representing the field inside and below the slab. However, considerations about the notion of effective media have shown that it is not correct to use the macroscopic notion of effective permittivity or effective permeability when evanescent waves propagate on the surface of a metamaterial with a transverse wavelength which approaches the period of the metamaterial. The limitation imposed by the heterogeneous structure of the material appears to be crucial. From this conclusion, it turns out that metamaterials could have very interesting focusing properties, provided that the elementary cell of the metamaterial is small enough. Finally, it has been shown that dielectric photonic crystals can provide an interesting solution for constructing left-handed materials in the visible and infrared regions. In order to know whether such a crystal can amplify the evanescent waves, we are actually trying to determine its permittivity and permeability from theoretical and numerical approaches.

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