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Self-Excited ac High Voltage Generation Using Water Droplets

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By letting water drops fall through rings into cans, high voltage can be spontaneously generated with no external electrical excitation. Previous work concerning this type of electric influence machine for dc and three-phase ac high voltage generation is extended to include multiphase, multifrequency operation by considering N streams and N cans. A distributed equivalent circuit representation is used to calculate the natural frequencies of the system, where it is found that many overstable modes are present. Experimental observations with up to five cans are presented. This device can serve as a model for phenomena concerned with atmospheric electricity.

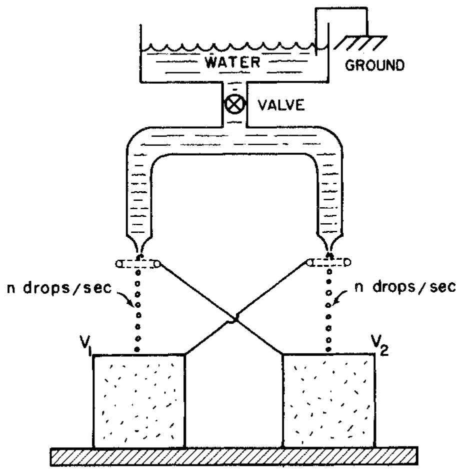


FIG. 1. Falling water drops spontaneously produce dc high voltage (10–20 kV) with no electrical inputs.

I. INTRODUCTION

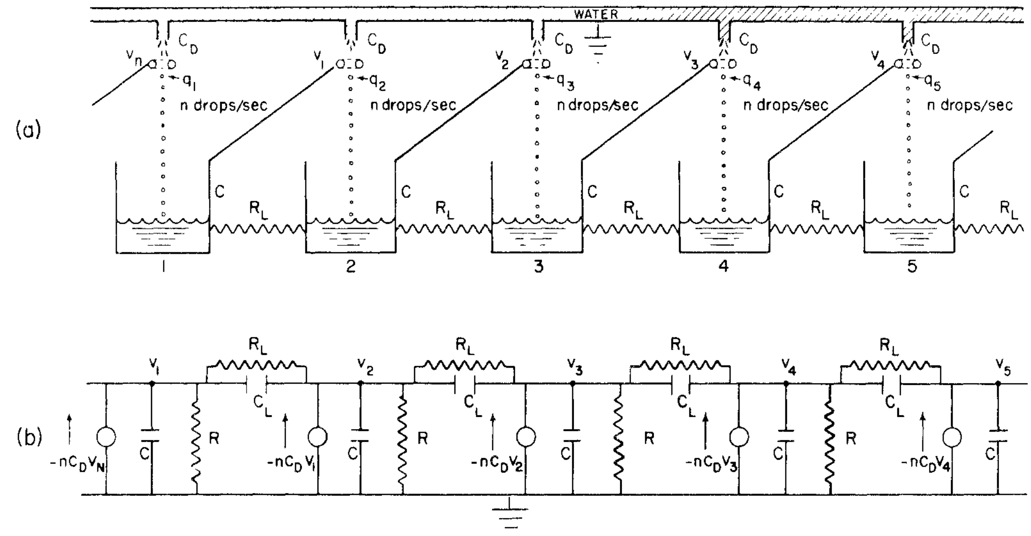
In 1867, Lord Kelvin[1] (then Sir W. Thomson) described his famous water dropper, where the falling of liquid drops was responsible for the generation of high voltages with no external electrical excitations. This electrohydrodynamic dynamo falls into the class of electrical influence machines similar to the classic Wimshurst machine, replacing the rotating metal disks by falling water drops. Kelvin used the dropper as a means of modeling atmospheric electricity since the presence of air, water, and high potential are the basic ingredients of a thunderstorm.

A similar apparatus is part of Moore's traveling electrostatic show, which Moore comments as being his most popular demonstration.[2] This device is also discussed and analyzed by Woodson and Melcher.[3] In their discussion, Lord Kelvin's device consists of two cans well insulated from one another, two pieces of wire and a pair of pipettes connected to a source of water as shown in Fig. 1. This apparatus spontaneously generates from 10–20 kV, which can be measured by a high impedance kilovoltmeter. The voltage builds up until there is electrical breakdown between the cans, or the electric attractive force deflects the drops until they hit the rings.

The generated voltage is maintained by the reciprocal arrangement, whereby each charge collector (the cans) is also the charge inducer (the rings) for the other. Any charge unbalance on the rings, either due to random fluctuations or perhaps to an initial charge purposely placed on the ring, will induce opposite charges on the stream falling through the ring. The resulting charged drops give up their charge to the can which then communicates this charge to the other inducer ring where the process is repeated, such that the net charge on the original inducer ring has been increased. This positive feedback results in the voltage build-up.

For voltage build-up to occur, the following conditions must be met;

1. The cans must be well insulated from one another, to minimize leakage currents.



$$V_i = \frac{R}{(RCs+1)}\left[-nC_D V_{i-1} + \frac{(V_{i-1} - V_i)(R_L C_L s+1)}{R_L} - \frac{(V_i - V_{i+1})(R_L C_L s+1)}{R_L}\right]$$

FIG. 2. (a) Each of the N identical cans and streams are cross connected to adjacent cans. The Nth can is coupled to the first stream, completing the loop. (b) The equivalent circuit representation shows that the potential of any node is related to the potentials of the preceding and succeeding nodes.

2. The stream must break into drops in the vicinity of the rings, to maximize the capacitive coupling between the rings and the forming droplets.

3. The liquid must be sufficiently conducting such that the induced charge relaxes to the surface of the stream before it breaks into drops. Any conducting liquid such as water meets this condition. In fact, even solid conducting balls can be used.

4. Once the drops are formed, the net charge on each drop must be conserved. The air must be sufficiently insulating such that its electrical relaxation time is much longer than the time required for a drop to reach the container below.

The two-can dropper so far discussed has voltage build-up at an exponential rate with no oscillations. A three-can dropper will produce self-excited three-phase ac high voltage. This version is included in the film by Melcher,[4] and is denoted as Euerle's dynamo, after the original inventor of this device.[5]

In addition to introducing many readers to the classic Kelvin dynamo and Euerle's ac modification, devices that this writer feels should be more widely known, it is the purpose here to generalize by considering "N cans" as a means of producing multiphase multifrequency ac high voltage. As a model of the atmosphere, a continuum of charge collectors and inducers can be imagined if we let $N \rightarrow \infty$ with the size of the cans becoming infinitesimally small. In processes such as electrostatic printing, paint spraying, and precipitation, an external high voltage source is needed. In configurations, similar to those presented here, the particles themselves can generate the necessary high voltage, eliminating the need of an external power supply.

In its simplicity, the analysis to be presented here yields a wealth of information. In the words of Kelvin: "The mathematical theory of the action . . . is particularly simple, but nevertheless curiously interesting."[1] Researchers in the area of atmospheric electricity should consider these interactions as fundamental building blocks in understanding more complicated phenomena. These self-excited dynamos illustrate how nature can arrange charge separation with no electrical driving forces. Perhaps such mechanisms are at work producing the known charge stratifications in clouds.[6]

Our approach will be to find an equivalent circuit representation for the system similar to that of Woodson and Melcher[3] consisting of distributed resistances, capacitances, and dependent sources. Since the differential equations which govern such systems are linear constant coefficient in time, exponential solutions of the form e^{st} can be assumed. To examine for stability, we simply solve for the natural frequencies s. If the real part of s is positive, the system is self excited, such that any perturbation will grow at an exponential rate. The imaginary part of s yields the oscillation rate of the resulting overstability.

For the special cases of $N=2$ and $N=3$, we will recover the results of Kelvin and Euerle.

II. EQUIVALENT CIRCUIT REPRESENTATION

Figure 2(a) illustrates the schematic configuration of N coupled cans and streams. Note that the Nth can is coupled to the first stream, completing the loop. For a net charge to be induced on falling droplets as they pass near a charged ring, the stream must break up in the immediate vicinity of the rings, as at this position, with the forming drop still tied to the main stream, charge can flow onto the droplet from the reservoir. The drops transport this charge to the can below which, because it is tied to the ring that encircles the next stream, induces charges on that stream also. This effect is successively transmitted to each stream, eventually reaching the initial stream which again finds charge induced such as to add to the initially induced charge. This regenerative feedback is the reason for the voltage build-up. It is important to note that the water reservoir remains neutral, as when charge is deposited upon a stream, an equal but opposite charge appears upon another stream.

If the drop is already dissociated from the stream as it passes near the ring, no net charge can be induced, as through the insulating air, no current can flow to deposit charge. If the stream breaks up into drops past the rings, the droplets are uncharged. If the stream just enters the cans without breaking into drops, it acts like a short circuit, keeping the cans at ground potential.

In deriving the equivalent circuit of Fig. 2(b), we consider in particular the drops falling into the ith can, where the induced charge on each drop is proportional to the voltage difference between the ring and the water in the pipette which is at ground potential,

$$q_i = -C_D v_{i-1}, \tag{1}$$

where the constant of proportionality C_D is the capacitance between the ring and the water droplet just as it breaks off from the stream. (There is no droplet *before* it breaks off, and hence no well defined capacity.) The minus sign is because image charges are induced on the stream. Because n drops/second fall into the can, the charge transport is modeled by a current source of value

$$nq_i = -nC_D v_{i-1}. \tag{2}$$

The cans as charge storers are represented as capacitors C to ground. The resistance R represents the leakage resistance to ground. The capacitance C_L represents the capacitance between adjacent cans plus the capacitance of a load, such as an electrostatic voltmeter. R_L represents leakage resistance between cans. The essential ingredients of self-excitation can be treated by a simpler idealized model with no losses where

$$C_L = 0,\quad R = \infty,\quad R_L = \infty, \tag{3}$$

but for generality we consider finite values for these parameters.

III. MATHEMATICAL ANALYSIS

At the ith node in the equivalent circuit of Fig. 2(b), the algebraic sum of the currents into the node must sum to zero, resulting in the general relation

$$V_i = \frac{R}{RCs+1}\left(-nC_D V_{i-1} + \frac{(V_{i-1}-V_i)}{R_L}(R_L C_L s+1) - \frac{(V_i - V_{i+1})}{R_L}(R_L C_L s+1)\right), \tag{4}$$

where since the elements of the equivalent circuit are linear time invariant, we have written the voltages in the form

$$v_1=V_1e^{st}$$
$$v_2=V_2e^{st}$$
$$\vdots$$
$$v_N=V_Ne^{st}.$$

Then the circuit equations of (4) can be put in the form

$$\begin{bmatrix} A & -D & 0 & 0 & 0 & \cdots & 0 & 0 & -B \\ -B & A & -D & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & -B & A & -D & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & -B & A & -D & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & -B & A & -D \\ -D & 0 & 0 & 0 & 0 & \cdots & 0 & -B & A \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ \vdots \\ V_{N-1} \\ V_N \end{bmatrix} = 0. \qquad (5)$$

where

$$A=1+[2R(R_LC_Ls+1)/R_L(RCs+1)],$$
$$B=[-nC_D+(R_LC_Ls+1)/R_L][R/(RCs+1)],$$
$$D=[(R_LC_Ls+1)/(RCs+1)](R/R_L).$$

Because all the rows are alike in form, the general difference equation for the ith row is

$$-BV_{i-1}+AV_i-DV_{i+1}=0. \qquad (6)$$

Equation (6) is a linear difference equation with constant coefficients, for which standard solutions can be assumed of the form[7]

$$V_i=K\lambda^i, \qquad (7)$$

which when substituted into (6) yields

$$-B+A\lambda-D\lambda^2=0, \qquad (8)$$

with solutions

$$\lambda_{1,2}=(A/2D)\pm[(A/2D)^2-(B/D)]^{1/2}. \qquad (9)$$

Equation (9) indicates that there are two characteristic solutions for (6). As with linear constant coefficient differential equations, the most general solution is the superposition of all allowed independent solutions,

$$V_i=K_1\lambda_1^i+K_2\lambda_2^i \qquad (\lambda_1\neq\lambda_2). \qquad (10)$$

The two conditions which (10) must obey are

$$V_0=V_N,$$
$$V_{N+1}=V_1, \qquad (11)$$

which can be checked in (6) with $i=1$ and $i=N$. Using (11) in (10), we obtain the coupled relations

$$K_1(1-\lambda_1^N)+K_2(1-\lambda_2^N)=0,$$
$$K_1\lambda_1(1-\lambda_1^N)+K_2\lambda_2(1-\lambda_2^N)=0, \qquad (12)$$

which for nontrivial solutions yields

$$\lambda_1^N=1; \qquad K_2=0,$$
$$\lambda_2^N=1; \qquad K_1=0. \qquad (13)$$

Either case in (13) can be treated simultaneously by using (9) to write

$$\lambda_{1,2}=(1)^{1/N}=(A/2D)\pm[(A/2D)^2-(B/D)]^{1/2}$$

or

$$(1)^{2/N}-(A/D)(1)^{1/N}+(B/D)=0.$$

Using the definitions of (5), we solve for the natural frequencies to be

$$s=\frac{[1-\exp(j2\pi r/N)]^2-\exp(j2\pi r/N)(R_L/R)-R_LnC_D}{R_L\{C\exp(j2\pi r/N)-C_L[1-\exp(j2\pi r/N)]^2\}};\qquad r=1,2,\cdots,N, \tag{14}$$

where we use the fact that

$$(1)^{1/N}=\exp(j2\pi r/N);\qquad r=1,2,\cdots,N. \tag{15}$$

From (13) we also obtain the relative phases of adjacent cans as

$$V_{n+1}/V_n=\exp(j2\pi r/N). \tag{16}$$

To examine the conditions for self-excitation and oscillation frequency, we must examine the real and imaginary parts of (14).

For the idealized model, when the lossless conditions of (3) hold, (14) reduces to

$$s=(-nC_D/C)\exp(-j2\pi r/N);\qquad r=1,2,\cdots,N. \tag{17}$$

IV. DISCUSSION OF RESULTS

$N=2$—Kelvin's Dynamo

Consider Kelvin's dynamo, for which $N=2$, then (14) and (16) yield

$$s_1=\frac{-4-(R_L/R)+R_LnC_D}{R_L(C+4C_L)};\qquad V_1=-V_2,$$

$$s_2=-[R^{-1}+nC_D]/C;\qquad V_1=V_2. \tag{18}$$

Note that the second root of (18) always decays, while the first root allows a growing solution with no oscillations if

$$nC_D>(4/R_L)+(1/R). \tag{19}$$

This indicates that if the leakage is significant, voltage buildup will not occur.

$N=3$—Euerle's Dynamo

Consider now Euerle's dynamo, for which $N=3$, in the limits given by (3). Then from (17)

$$s_1=(-nC_D/C)(-\tfrac{1}{2}+\tfrac{1}{2}\sqrt{3}j);\qquad V_3/V_2=V_2/V_1=\exp(j4\pi/3),$$

$$s_2=(-nC_D/C)(-\tfrac{1}{2}-\tfrac{1}{2}\sqrt{3}j);\qquad V_3/V_2=V_2/V_1=\exp(j2\pi/3),$$

$$s_3=-nC_D/C;\qquad V_1=V_2=V_3. \tag{20}$$

The first two roots which are complex conjugates represent overstability while the third root strictly decays. Note the three phase relationship between the voltages for the growing modes. If leakage became significant, a condition similar to (19) would be necessary for voltage build-up to occur.

N Arbitrary

In general, for an arbitrary number of cans, there are many growth rates and oscillation frequencies. Since random fluctuations will excite all modes, that mode with the fastest growth rate will dominate. We restrict our discussion now to the lossless case with conditions given by (3), so we focus attention on (17).

If we have an even number of cans, note from (17) that the fastest growing mode occurs when

$$r=\tfrac{1}{2}N; \qquad s=nC_D/C; \qquad V_{n+1}/V_n=-1. \quad (21)$$

This mode is purely exponential with no oscillations, and the voltages alternate in a positive–negative sequence between adjacent cans. However, if we measure the potential difference between alternate cans, the potential difference due to this fastest growing mode will be zero, thus allowing measurement of oscillatory modes with slower growth rates. With an odd number of cans, the fastest growing modes will be for

$$r=\tfrac{1}{2}(N\pm1); \qquad s=(nC_D/C)\exp[\mp j(\pi/N)];$$
$$V_{n+1}/V_n=-\exp[\pm j(\pi/N)], \quad (22)$$

with frequency of oscillation

$$\omega_0=(nC_D/C)\sin(\pi/N). \quad (23)$$

V. EXPERIMENTAL OBSERVATIONS

A four-can version was built, depicted in Fig. 3. The maximum voltage build-up between cans was in the range of 10–20 kV. Because of the valves, it was possible to operate with either $N=2$, $N=3$, or $N=4$. Voltage build-up was measured with an electrostatic kilovoltmeter and could be observed visually by the spreading of the drops. If electrical breakdown did not occur, the voltage was limited by the attractive force on the drops on the rings. At limiting voltages, the drops would spiral about the rings.

With strict exponential growth with no oscillations, the drops would spread and then spiral in a steady-state fashion until voltage breakdown occurred, and then the cycle would begin again.

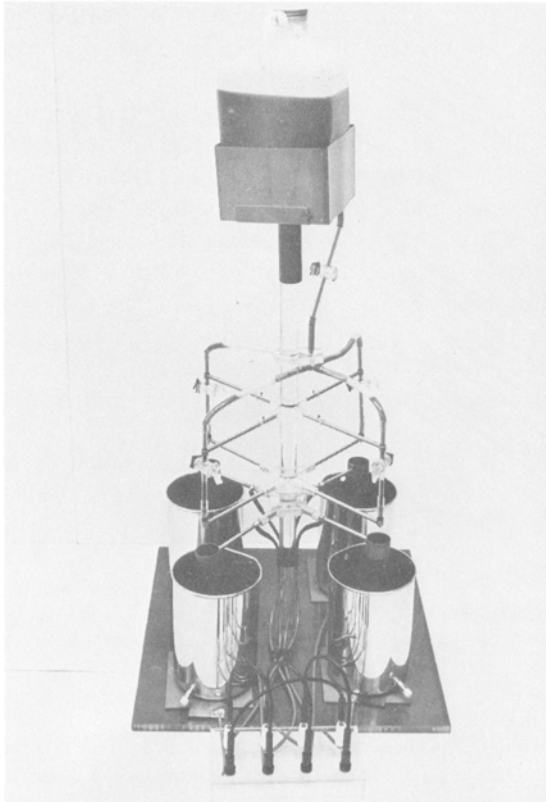


FIG. 3. By adjusting the valves this four-can version can be operated with either $N=2$ (dc), $N=3$ (three-phase ac), or $N=4$ (two-phase ac or dc).

With three cans, each stream would spread, spiral, and contract in a three phase 120° sequence. For comparison, an identical fifth can and stream was added to the device in Fig. 3. The ratio of frequencies for the five-can version to that of three cans, assuming the geometry and drop rate are the same, are given by the imaginary part of (17) for those roots which are in the right-half s plane, also given by (23)

$$f_5/f_3=\sin 36^\circ/\sin 60^\circ=0.68. \quad (24)$$

It was measured that $f_5=(1/184)\ \mathrm{sec}^{-1}$ and $f_3=(1/125)\ \mathrm{sec}^{-1}$ yielding the ratio

$$(f_5/f_3)\ \text{measured} \approx 0.68 \quad (25)$$

in agreement with (24).

Actual values for C_D and C are difficult to measure because of the irregular geometry, but it is safe to assume these values to be in the low picofarad range. Typically the number of drops falling per second was on the order of 10 ($n=10$), as measured by a strobotac. Thus from (23) and the measured frequencies, this device has $C_D<C$. It is also difficult to measure the resistances R and R_L as these values are usually due to leakage from moisture and dirt, as well as the resistance due to the supporting structure. However, if we assume $C_D \approx 10$ pf with $n=10$, the leakage resistance must exceed 10^{10} Ω, as determined from (19).

With four cans, the natural frequencies are either pure real or pure imaginary. Experimentally, the drops would behave in the same steady spiraling manner as for two cans, indicating the dominance of the pure exponential mode. However, the voltage difference between any two opposite cans (cans one and three, or two and four) was oscillating as expected.

A bigger three-can dynamo was built using 20 gallon cans, similar to that shown in Melcher's film.[4] Here voltages were in the 30 kV range with frequencies on the order of 1 Hz.

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