Does the glory have a simple explanation?

H. M. Nussenzveig*

NASA Goddard Space Flight Center, Greenbelt, Maryland 20771

Received April 16, 2002

The explanation for the meteorological glory provided by the complex angular momentum theory is revisited in response to comments that a simpler physical picture would be desirable. New results that confirm the tunneling origin of the glory and the roles of resonances and surface waves in this phenomenon are presented, and expressions for averaged angular distribution and polarization features are given.

OCIS codes: 290.4020, 290.1350, 260.1960, 010.1290.

The beautiful meteorological phenomenon of the glory arises from backscattering of sunlight by cloud water droplets[1] with typical size parameter x (circumference/wavelength) of the order of 10^2. The Mie series solution requires a number of terms of this order to represent the effect. Geometrical optics is totally inadequate (see below). Is there a simple physical explanation? Whereas simplicity, like beauty, may be in the eye of the beholder, it would be hard to argue that the Mie series provides a simple physical picture.

An attempted explanation in terms of surface waves was suggested by van de Hulst,[1] who also proposed a parameterization for the angular distribution of the glory rings. However, he was unable to predict the values of the surface wave and angular parameters.

An asymptotic approximation to the Mie series based on complex angular momentum (CAM) theory[2] led to a new theory of the glory,[3] which determined the van de Hulst surface wave parameters and showed that several additional effects must be taken into account. A detailed presentation of the theory, which accounts for all known features of the glory, can be found elsewhere.[2]

Two basic ingredients in the CAM theory of the glory are the Debye expansion and resonances. The Debye expansion represents the interaction of the incident wave with the sphere in terms of direct reflection and an infinite series of internal reflections after penetration inside the sphere, much like the multiple-reflection treatment of the Fabry–Perot interferometer. The van de Hulst surface wave is associated with the third Debye term (one internal reflection).[2] Resonances appear as sharp spikes in all Mie quantities plotted as functions of x (ripple fluctuations). They are so prominent and ubiquitous in backscattering that it has been proposed[4] that they are the primary source of the backscattered intensity. The highly complex ripple pattern has been spectrally analyzed,[5] revealing a quasi-periodic character, with a fundamental period of ~ 0.8 and another prominent period of ~ 14. CAM theory provides a physical model for the origin of resonances and allows one to determine their positions and widths with high accuracy.[6]

However, an attempted comparison[7] between the backscattered intensity, averaged over the fundamental quasi-period, and the main nonresonant CAM theory terms, through a broad range of x, found a substantial order-of-magnitude discrepancy. Furthermore, several authors[8–10] have complained that CAM theory is complicated, not providing a simple prescription for the glory angular pattern, and they have wondered whether a simple physical explanation can be found.

Here we address these alleged shortcomings of CAM theory. We restrict our consideration to the range of x most often found in natural glories,[1] not exceeding a couple of hundred. Within this range, according to CAM theory[2] the dominant physical effects in backscattering arise from the third Debye term (which contains the van de Hulst surface wave) and from resonances, besides smaller contributions from several other terms. Furthermore, these effects are produced mainly by incident rays traveling outside the droplet, coupled to the inside by tunneling.

"Tunneling" is here a synonym for evanescent wave coupling, as occurs in frustrated total reflection. This terminology is justified by the well-known analogy between optics and mechanics: For each partial wave, Mie scattering is associated with an equivalent potential, an attractive well surrounded by a centrifugal barrier.[2] This tunneling picture has been experimentally verified by use of evanescent coupling to excite resonances.[11]

To assess the validity of CAM physical interpretations we plot, for the special[2] refractive index $N = 1.33007$, different contributions to the backward gain G, the ratio[1] of the backscattered intensity to that for an ideal isotropic scatterer (i.e., a totally reflecting sphere in the geometrical optics limit). In Fig. 1 we compare, for $100 \le x \le 200$, the contribution G_2 to G from the third Debye term, computed from the Mie series, with its lowest-order CAM asymptotic approximation $G_{2\mathrm{CAM}}$,[2] the sum of the van de Hulst surface wave and the geometrical-optics (WKB) once internally reflected axial ray amplitude. The oscillations, which arise from interference between these two contributions, have a period of ≈ 14. We see that $G_{2\mathrm{CAM}}$ is already a fair approximation of the exact Mie–Debye result, and its physical interpretation is verified.

Next, we probe the relative weights of different incident ray domains in backscattering, with the help of the localization principle,[1] according to which, for $x \gg 1$, the lth partial wave in the Mie series is

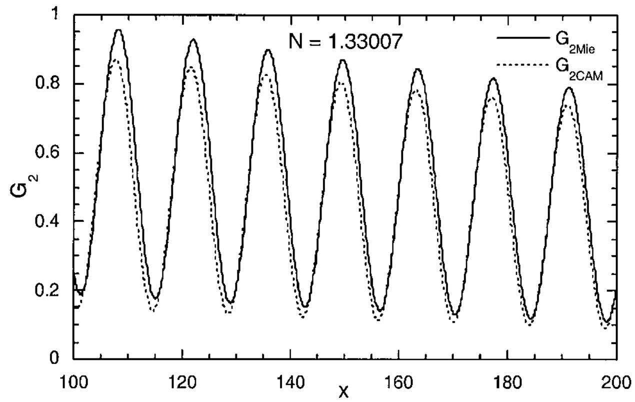

Fig. 1. Comparison of Mie and CAM theories for the third Debye term.

associated with an impact parameter $b_l = (l + 1/2)/k$ (k is the wave number). We call contributions with $b_l > a$, corresponding to incident rays that pass outside the droplet and interact with it by tunneling inside, above-edge contributions; below-edge rays, with $b_l < a$, hit the droplet. We plot the running average of the gain, $\langle G \rangle$, averaged over the fundamental quasi-period $\delta x = 0.82$.

In Fig. 2, $\langle G_{\mathrm{Mie}} \rangle$ is the Mie result, $\langle G_{\mathrm{ae}} \rangle$ is the above-edge contribution, and $\langle G_{\mathrm{be}} \rangle$ is the below-edge contribution. The geometrical-optics approximation, $\langle G_{\mathrm{go}} \rangle$, is clearly unable to account for the results. The below-edge contribution has a complicated structure that arises from interference among many near-peripheral higher-order Debye terms.[2] We see that the above-edge tunneling contribution is indeed strongly dominant, although interference with the below-edge terms remains appreciable.

Also plotted in Fig. 2 is the nonresonant CAM approximation $G_{\mathrm{CAM,nr}}$, which excludes resonance contributions but includes, besides $G_{2\mathrm{CAM}}$, the 10th-order rainbow shadow term[2] (this term, which is almost negligible within the plotted range, becomes dominant for $x \geq 10^3$). Even though $G_{\mathrm{CAM,nr}}$ amounts typically to less than half of the total gain, strikingly, it governs the qualitative behavior of $\langle G_{\mathrm{Mie}} \rangle$, with close correlation between peaks and valleys in the two curves. I have verified that $G_{\mathrm{CAM,nr}}$ is also dominated by above-edge contributions, so tunneling has a major role as well in forming the van de Hulst surface wave. This surface wave has been experimentally detected in the terahertz domain.[12] The remaining (large) contribution to $\langle G_{\mathrm{ae}} \rangle$ is due to resonances, which are also accurately represented by CAM theory.[2] Thus, in agreement with CAM theory, tunneling is dominant in producing the glory.

An earlier attempt to perform a similar comparison[7] failed because it misrepresented the van de Hulst surface wave intensity, reducing it by approximately 1.5 orders of magnitude, and it did not take into account interference with the once internally reflected axial-ray contribution.

What about the angular distribution and polarization? For near-backward scattering, CAM theory yields the following approximation[2] for the scattering amplitudes:

$$S_1(x, \theta) \approx 2S^M(x)J_1'(u) + 2S^E(x)[J_1(u)/u], \quad u \equiv (\pi - \theta)x \gg 1, \quad (1)$$

where S^M (S^E) is the magnetic (electric) multipole contribution to the Mie series at $\theta = \pi$; by interchanging $S^M \leftrightarrow S^E$ we get $-S_2(x, \theta)$. A similar expression was proposed by van de Hulst,[1] who tried to adjust the Bessel function coefficients by an empirical fit to glory ring size observations. It follows from expression (1) that, for natural incident light, the angular distribution and polarization of the near-backward intensity are given, respectively, by

$$i(x, \theta)/i(x, \pi) = J_0{}^2(u) + c(x)J_2{}^2(u), \quad (2)$$

$$P(x, \theta) = 2\sqrt{c(x)}\,J_0(u)J_2(u)/[J_0{}^2(u) + c(x)J_2{}^2(u)], \quad (3)$$

where

$$c(x) \equiv |[S^E(x) - S^M(x)]/[S^E(x) + S^M(x)]|^2. \quad (4)$$

The parameter $c(x)$ undergoes ripple fluctuations as large and rapid as those of $G(x)$, so angular distribution and polarization for an artificial droplet monodispersion would be highly sensitive to size. The running average $\langle c(x) \rangle$ over $\delta x = 0.82$, plotted in Fig. 3, also shows substantial variations, which are reflected in the pattern variability of natural glories: There is no unique glory angular distribution. A roughly representative value, in the upper size range of Fig. 3, might be taken as $\langle c \rangle \approx 3$, which is consistent with van de Hulst's empirical fits.[1] The corresponding angular distribution and polarization, plotted in Fig. 4, agree with the observed haziness of the first dark glory ring and strong radial polarization of the outer rings, opposite that of the central ring. Note that the polarization reversals occur in regions of near-vanishing intensity.

To sum up: Near-peripheral, mainly external, incident light produces the glory. Resonances and

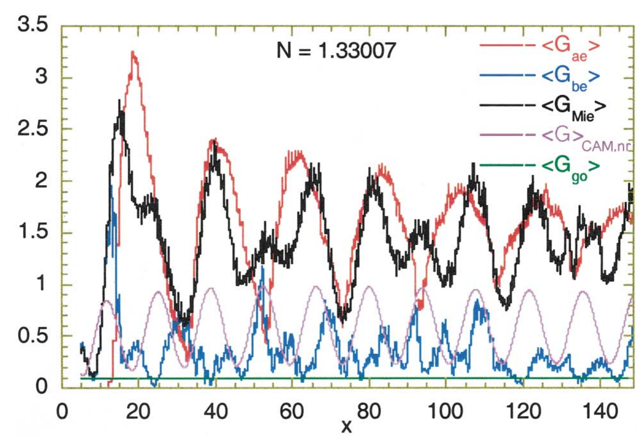

Fig. 2. Running average, over $\delta x = 0.82$, of the backscattering gain. Black curve, Mie result; red curve, above edge; blue curve, below edge; green curve, geometrical optics; lavender curve, nonresonant CAM result.

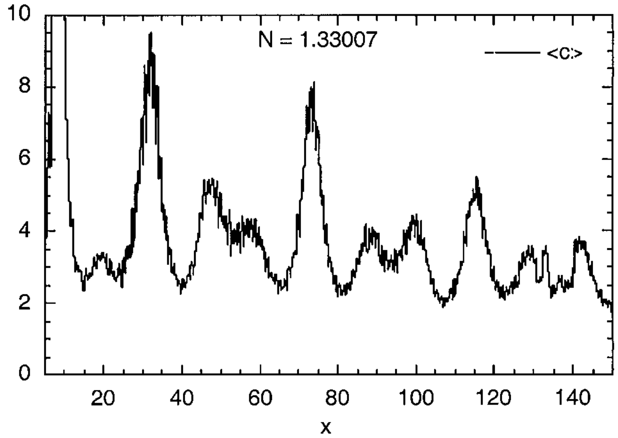

Fig. 3. Running average, over $\delta x = 0.82$, of parameter $c(x)$.

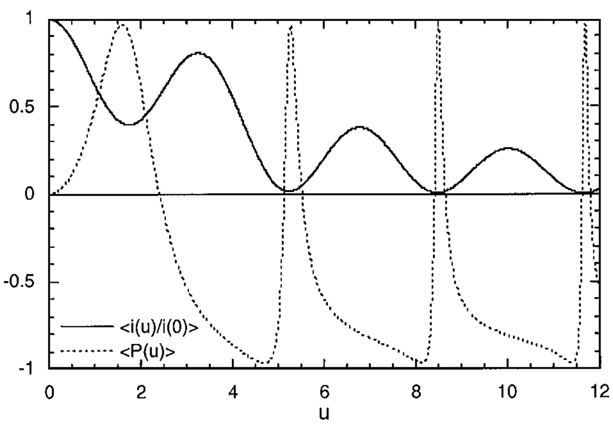

Fig. 4. Typical angular distribution and polarization in natural glories.

van de Hulst's surface waves are the main contributors to natural glories. Spherical symmetry enhances these contributions by axial focusing.[1,2] Mie theory describes the glory by the sum of a large number of complicated terms within which the physical mechanisms cannot be discerned. CAM theory brings out the dominant physical effects and provides an accurate representation for each of them. That it does so by analytic continuation seems inevitable: I know of no other way of quantitatively representing tunneling. The glory exemplifies sensitive dependence on initial conditions in scattering. Its appearance can change during a single observation. A sufficiently large spheroidal shape deformation can lead to chaotic behavior.[13]

How robust are the above results with respect to possible water droplet deformations? It has been shown[14] that small surface perturbations (which might arise, e.g., from capillary waves) not only can significantly reduce resonance Q factors but may also modify the resonance excitation mechanism, favoring coupling through surface scattering. In the glory size parameter range, this modification occurs only for high-Q values (in excess of 10^6); lower-Q resonances are not affected. However, high-Q resonances ($Q > 10^5$) do not contribute appreciably to the running averages plotted in Figs. 2 and 3, because the peak resonant contribution to G is bounded above (by $4N^2$), so the area contributed by each resonance varies inversely with its Q. Thus the results described above are not affected.

In conclusion, in answer to the question posed by the title of this paper: It would be foolhardy to exclude the possibility of future simplification, but one should bear in mind Einstein's admonition that "Everything should be made as simple as possible, but not simpler."

This research was performed while the author held a National Research Council Research Associateship Award at NASA Goddard Space Flight Center. The author's e-mail address is moyses@if.ufrj.br.

*Permanent address, Instituto de Física, Universidade Federal do Rio de Janeiro, Caixa Postal 68528, 21945-970 Rio de Janeiro, R.J., Brazil.

References

1. H. C. van de Hulst, *Light Scattering by Small Particles* (Wiley, New York, 1957).
2. H. M. Nussenzveig, *Diffraction Effects in Semiclassical Scattering* (Cambridge U. Press, Cambridge, 1992), and references therein.
3. V. Khare and H. M. Nussenzveig, Phys. Rev. Lett. **38,** 1279 (1977).
4. J. R. Probert-Jones, J. Opt. Soc. Am. A **1,** 822 (1984).
5. S. T. Shipley and J. A. Weinman, J. Opt. Soc. Am. **68,** 130 (1978).
6. L. G. Guimarães and H. M. Nussenzveig, J. Mod. Opt. **41,** 625 (1994).
7. R. G. Pinnick, S. G. Jennings, P. Chylek, C. Ham, and W. T. Grandy, Jr., J. Geophys. Res. **88,** 6787 (1983).
8. R. Greenler, *Rainbows, Halos, and Glories* (Cambridge U. Press, Cambridge, 1980), p. 146.
9. C. F. Bohren and D. R. Huffman, *Absorption and Scattering of Light by Small Particles* (Wiley, New York, 1983), p. 389.
10. D. K. Lynch and W. Livingston, *Color and Light in Nature*, 2nd ed. (Cambridge U. Press, Cambridge, 2001), p. 136.
11. L. Collot, V. Lefèvre-Seguin, M. Brune, J. M. Raimond, and S. Haroche, Europhys. Lett. **23,** 327 (1993).
12. R. A. Cheville, R. W. McGowan, and D. Grischkowsky, Phys. Rev. Lett. **80,** 269 (1998).
13. A. Mekis, J. U. Nöckel, G. Shen, A. D. Stone, and R. K. Chang, Phys. Rev. Lett. **75,** 2682 (1995).
14. J. P. Barton, J. Opt. Soc. Am. A **16,** 1974 (1999). I thank an anonymous referee for drawing my attention to this reference.