Theoretical and numerical study of gratings consisting of periodic arrays of thin and lossy strips

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We start with the study of gratings consisting of periodic arrays of thin lossy strips with arbitrary cross sections. Then we investigate the behavior of such gratings if they are sandwiched between multilayered structures. Taking advantage of theoretical considerations recently published, we propose an approximate method and stress the numerical aspect. The resulting computer code seems interesting mainly for the study of gratings in the far-infrared and microwave ranges.

1. INTRODUCTION AND NOTATIONS

Throughout the paper we assume that the grating spacing d is of the same order of magnitude as the wavelength in vacuum λ_0. An efficient method has been proposed for perfectly conducting wire gratings,[1,2] but, in our opinion and except maybe for lamellar gratings,[3–5] only the differential method[6] has been used to solve rigorously the problem of transmission gratings having a finite conductivity. This method, which leads to a differential system of coupled differential equations, can easily be implemented on big computers. Nevertheless, those who wish to use microcomputers certainly would like to have a simpler and faster method at their disposal, especially if they foresee the study of bidimensional gratings for which the computation time is a worrisome problem. We propose such a method that applies to the case of highly conducting gratings whose thickness is small compared to the wavelength (in a sense that will be specified below). This frequently happens in the far-infrared or in the microwave range (suppose, for instance, that the groove depth h is some tens of a millimeter and λ_0 some centimeters or decimeters). For clarity, we have to recall and comment on some theoretical results already published (Section 2) or to be published (Section 5). These results are of prime importance for a good understanding of the paper and especially for Sections 5 and 6, which indeed contain the more important and new ideas.

$Oxyz$ is an orthogonal coordinate system. We deal only with time harmonic and z independent fields in TE or TM polarization (which means that the electric field or the magnetic field is parallel to the z axis). The total field is represented by a scalar function $u(x, y)$ taking into account a time dependence in $\exp(-i\omega t)$. This function is the z component of **E** or **H** depending on the polarization (TE or TM). We denote by ϵ_0 and μ_0 the permittivity and the permeability of vacuum.

2. SOME THEORETICAL PREREQUISITES

Figure 1a shows a wire metallic grating G_{h_0} with a pitch d and thickness h_0 surrounded by vacuum.[7] We suppose that the metal has the same permittivity and permeability as vacuum and a real conductivity σ (a model often used in the far-infrared and microwave ranges). Under these conditions the metal is equivalent to a lossy dielectric with relative permittivity $\epsilon = 1 + i\sigma/\epsilon_0\omega$. The cross section of a wire can be described by a positive function $f(x)$ whose maximum value is unity; $f(x)$ defined on the period interval $(-d/2, d/2)$ vanishes for $c/2 < |x| < d/2$, and the cross section is limited by the x axis, the lines $x = \pm c/2$ and the graph of $y = h_0 f(x)$.

Let us consider (Fig. 1b) a family of gratings G_h whose thickness h tends to zero and whose conductivity varies in such a way that the product of the thickness and the conductivity keeps a constant value. This means that we assign to the grating G_h a conductivity $\sigma h_0/h$. If illuminated by a given incident field $u^i(x, y)$, this grating gives rise to a diffracted field $u_h{}^d(x, y)$ corresponding to a total field $u_h(x, y) = u^i(x, y) + u_h{}^d(x, y)$.

If h vanishes, G_h reduces to the infinitely thin grating G_0 (Fig. 1c), and $u_h(x, y)$ tends to a limit field $u_0(x, y)$ that has been studied in detail in a previous theoretical paper.[8] Hereafter G_0 will be called a grating interface. The limit field $u_0(x, y)$ verifies the Helmholtz equation in the complementary of G_0 and satisfies on G_0 boundary conditions[8] that are neither the Dirichlet condition nor the Neumann condition. More precisely, if $u_0{}^+(x)$ and $u_0{}^-(x)$ are the limits of $u_0(x, y)$, where y tends to zero by positive or negative values, respectively, and if $k_0 = \omega(\mu_0\epsilon_0)^{1/2}$, the pertinent boundary conditions[8] depend on the function f and on a dimensionless parameter s associated with G_{h_0} and defined as

$$s = h_0 \sigma \eta_0, \tag{1}$$

where $\eta_0 = (\mu_0/\epsilon_0)^{1/2}$ is the vacuum impedance. These conditions, which of course also depend on the polarization, are

$$u_0{}^+ = u_0{}^-, \qquad \left(\frac{\partial u_0}{\partial y}\right)^+ - \left(\frac{\partial u_0}{\partial y}\right)^- = -ik_0 s f u_0{}^+ \quad \text{in the TE case,} \tag{2}$$

$$\left(\frac{\partial u_0}{\partial y}\right)^+ = \left(\frac{\partial u_0}{\partial y}\right)^-, \qquad u_0{}^+ - u_0{}^- = \frac{is}{k_0}\left(\frac{\partial u_0}{\partial y}\right)^+ f \quad \text{in the TM case.} \tag{2'}$$

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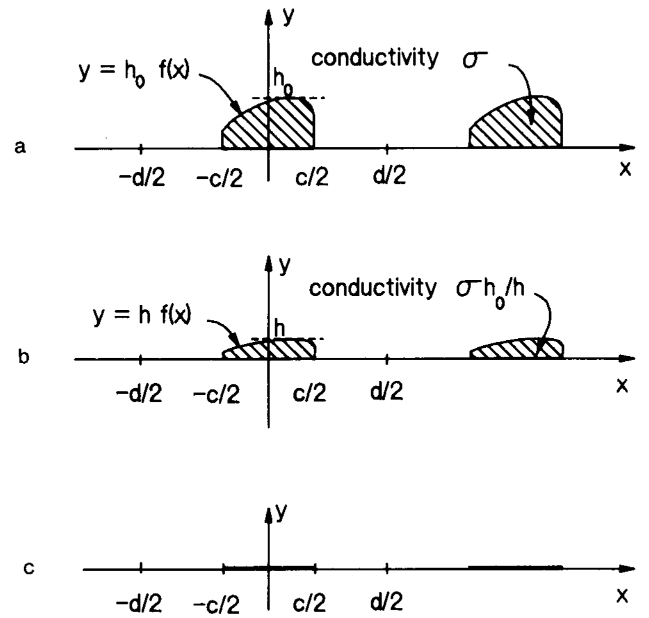

Fig. 1. Grating G_{h_0}. The hatched area is the cross section of a wire. b, Grating G_h. We consider G_h in the course of the limiting process. c, Heavy line, the infinitely thin grating G_0.

Maybe it is better to say that Eqs. (2) and (2′), which give the jumps of u_0 and its normal derivative at $y = 0$, are transmission conditions rather than boundary conditions, which we will consider from now on.

It must be noticed that Eqs. (2) and (2′), respectively, reduce to the Dirichlet boundary condition ($u_0{}^+ = u_0{}^- = 0$) and to the Neumann boundary condition $[(\partial u_0/\partial y)^+ = (\partial u_0/\partial y)^- = 0]$ for only perfectly conducting wires ($s = \infty$). Of course these conclusions are reminiscent of considerations developed by several authors for rectangular cross sections[9–11] and linked with the concept of surface impedance. For example, the reciprocal of our parameter s is nothing other than the normalized square resistance used in two recent papers.[10,11] Nevertheless we want to emphasize that the paper that we refer to[8] is founded on the sound basis of modern functional analysis; it states precisely which notion of convergence must be used and, more generally, tries to throw light on the concept of infinitely conducting and infinitely thin material. It must be also emphasized that the transmission conditions [Eqs. (2) and (2′)] depend on $f(x)$, i.e., on the initial shape of the cross section (a point that, to our knowledge, has not been mentioned before). Indeed all the proofs in Ref. 8 are related to a single rod, but, no doubt, the generalization to a grating would be straightforward for a mathematician and intuitive for a physicist.

3. APPLICATION TO THE THEORY OF THIN AND HIGHLY CONDUCTING METAL WIRE GRATINGS

Until now we have been concerned with a rather academic problem. Let us turn to numerical and more practical considerations. A given metal grating G_{h_0}, defined by h_0, σ, and $f(x)$, is illuminated in vacuum by a plane wave under the incidence θ:

$$u^i(x, y) = \exp[ik_0(x \sin\theta - y \cos\theta)]. \qquad (3)$$

We look for the associated total field $u_{h_0}(x, y)$. This problem can be solved by the differential method as explained in Ref. 6. But if h_0 is small enough we can expect that $u_{h_0}(x, y)$ is not extremely different from the limit field $u_0(x, y)$ defined in Section 2. This guess has been checked with the help of a computer code based on the differential method that, for small thicknesses, appears to be perfectly reliable whatever the polarization. Starting from the grating G_{h_0} (thickness h_0, conductivity σ), let us consider again the family of gratings G_h described in Section 2 (thickness h, conductivity $\sigma h_0/h$) and corresponding to the parameter $s = \sigma h_0 \eta_0$. For the grating G_h, let $e(h)$ be the sum of the $e_n(h)$, $e_n(h)$ being the efficiency in the nth diffraction order. Consequently, the fraction $Q(h)$ of the incident energy absorbed by the Joule effect is $1 - e(h)$. In Fig. 2, obtained by using the differential method, one curve corresponds to a family of gratings G_h and gives $e(h)$ versus $\log(k_0 h)$. It appears that, provided that $k_0 h$ is less than 10^{-2} (and even less than 1/3 for $s = 0.5$), $e(h)$ is practically constant and equal to the value $e(0)$ (associated with G_0) that can be obtained in a much simpler way as explained below in Section 4. Indeed, we have verified that this conclusion also holds for each of the efficiency e_n and for other cross sections. In other terms and provided that $k_0 h_0 < 10^{-2}$, the gratings G_{h_0} and G_0 have the same behavior; from the numerical point of view we can therefore replace the study of the grating G_{h_0} with the study of grating G_0. Obviously the properties of the grating G_{h_0} depend on the shape of its cross section, and this is why the function f must appear in Eqs. (2) and (2′).

4. NUMERICAL STUDY OF THE INFINITELY THIN GRATING G_0 SURROUNDED BY A VACUUM

This is indeed a simple matter, and we will give only an outline.[12] As we did for classical gratings,[6] it can be proved that the total field $u(x, y)$ associated with the incident field $u^i(x, y)$ can be represented by a Rayleigh expansion for $y > 0$ and for $y < 0$ as well:

$$u(x, y) = u^i(x, y) + \sum_{n=-\infty}^{+\infty} R_n \exp(i\alpha_n x + i\beta_n y) \text{ for } y > 0,$$

$$\text{for } y > 0, \qquad (4)$$

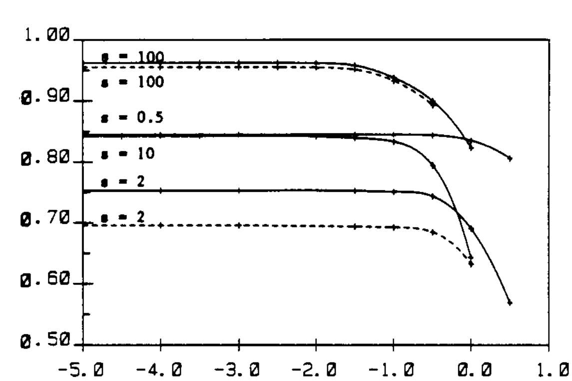

Fig. 2. We suppose that $c = d/2$, $\lambda_0/d = 0.75$, $\theta = 0$, and we deal with a rectangular cross section: $f(x) = 1$ if $|x| < c/2$ and 0 if $c/2 < |x| < d/2$. Each curve shows, for a given value of parameter s, the variations of e versus $\log(k_0 h)$. Solid curves and dashed curves correspond, respectively, to TE and TM polarization.

$$u(x, y) = \sum_{n=-\infty}^{+\infty} T_n \exp(i\alpha_n x - i\beta_n y) \qquad \text{for } y < 0, \tag{5}$$

with

$$\alpha_n = k_0 \sin\theta + n2\pi/d, \qquad \beta_n^{\,2} = k_0^{\,2} - \alpha_n^{\,2}, \qquad \beta_n \text{ or } \beta_n/i > 0. \tag{6}$$

In order to find the Rayleigh coefficients R_n and T_n, we have to write that $u(x, y)$ fulfills the transmission conditions [Eqs. (2) or (2′)]. To this end it is convenient to introduce the functions

$$u_n^{+}(y) = \exp(+i\beta_n y), \qquad u_n^{-}y = \exp(-i\beta_n y), \tag{7}$$

which permits us to rewrite Eqs. (4) and (5) by using a generalized Fourier series[6]

$$u(x, y) = \sum_{n=-\infty}^{+\infty} [\delta_{n,0} u_n^{-}(y) + R_n u_n^{+}(y)]\exp(i\alpha_n x) \qquad \text{for } y > 0, \tag{4′}$$

$$u(x, y) = \sum_{n=-\infty}^{+\infty} T_n u_n^{-}(y)\exp(i\alpha_n x) \qquad \text{for } y < 0. \tag{5′}$$

Recall that if a periodic function f and a pseudoperiodic function g are written as

$$f = \sum_n f_n \exp\left(i\,\frac{n2\pi}{d}\,x\right), \qquad g = \sum_n g_n \exp(i\alpha_n x),$$

then their product $p = fg$ can be expanded in a generalized Fourier series

$$p = \sum_n p_n \exp(i\alpha_n x), \qquad \text{with } p_n = \sum_m f_{n-m} g_m.$$

Therefore the matching of Eqs. (4′) and (5′) can be easily performed by writing, for each transmission condition, that the two members have the same Fourier coefficients. In TE polarization we are led to

$$R_n - T_n = -\delta_{n,0}, \tag{8}$$

$$R_n + \sum_m \left(\delta_{m,n} + k_0 \frac{s}{\beta_n} f_{n-m}\right) T_m = \delta_{n,0}, \tag{8′}$$

whereas in TM polarization we have

$$R_n + T_n = \delta_{n,0}, \tag{9}$$

$$R_n - \sum_m \left(\delta_{m,n} + \frac{s}{k_0} \beta_m f_{n-m}\right) T_m = -\delta_{n,0}. \tag{9′}$$

Keeping only $P = 2N + 1$ terms in the Fourier series, we get R_n and T_n by solving the linear systems [Eqs. (8) and (8′) or Eqs. (9) and (9′)].

As an illustration, curves are given for two profile functions described in Fig. 3. These functions correspond to a rectangular or a triangular cross section for grating G_{h_0}. These curves (Figs. 4–7) show, for $\theta = 30°$, $|R_0|$ versus d/λ_0 for different values of c/d. They have been obtained with $s = 0.24\pi$, a value that corresponds to $R = 500\ \Omega$/square in the terminology of Ref. 10. Figure 4 shows that our results are in perfect agreement with those given by Hall and Mittra[10] in their Fig. 6, except perhaps near a Rayleigh anomaly, for which the shape of the computed curve is of course strongly dependent on the number of sampling points. Our Figs. 4

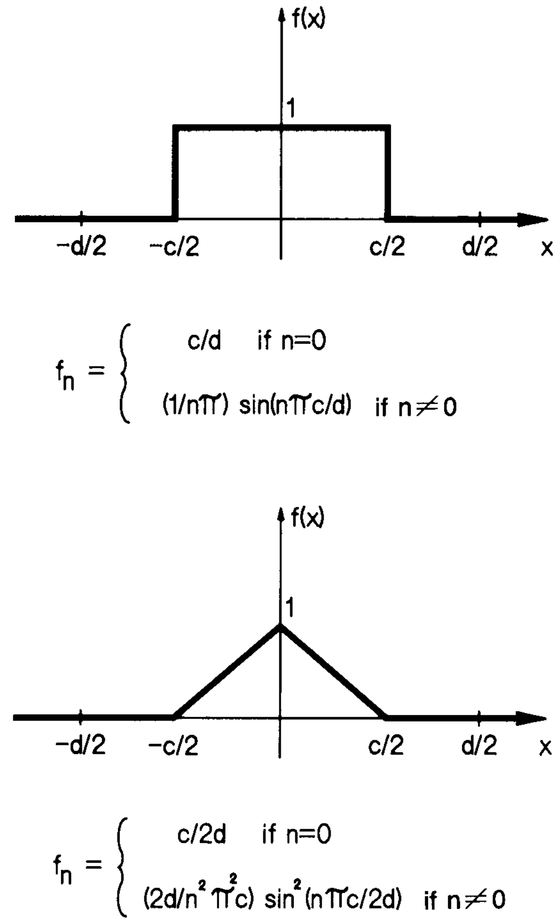

Fig. 3. Two profile functions represented on one period and their Fourier coefficients.

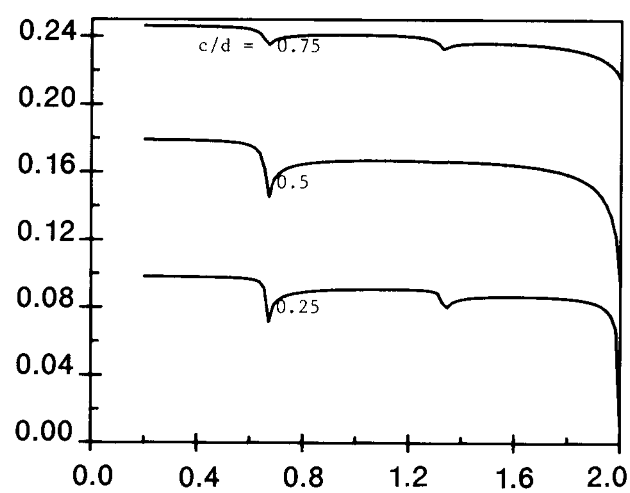

Fig. 4. Rectangular cross section, TE polarization.

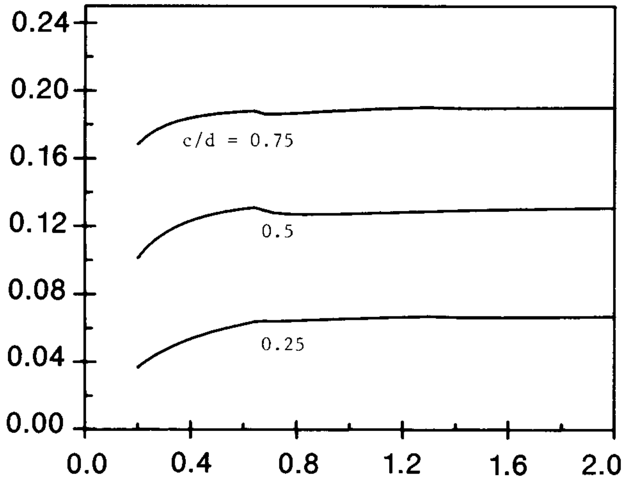

Fig. 5. Rectangular cross section, TM polarization.

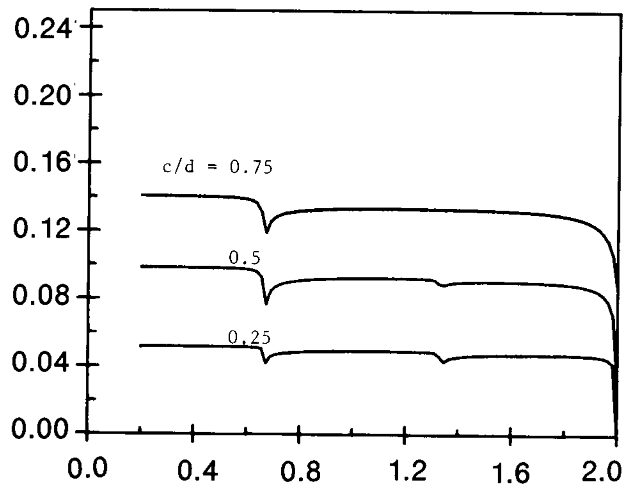

Fig. 6. Triangular cross section, TE polarization.

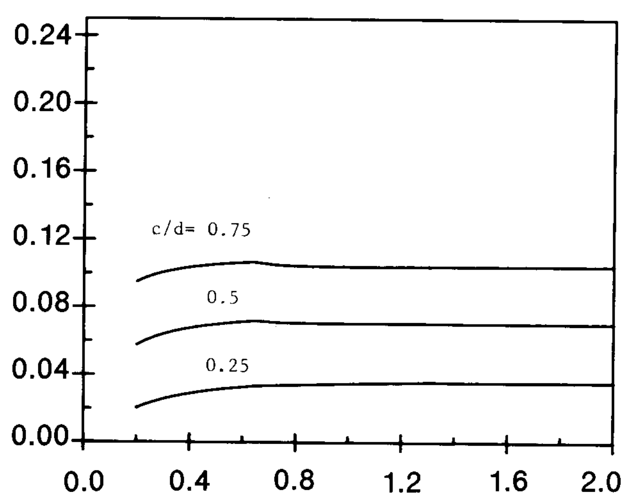

Fig. 7. Triangular cross section, TM polarization.

and 5, as compared with our Figs. 6 and 7, show the influence of the profile function $f(x)$. Only a few seconds are needed to obtain one curve (Fig. 4–7) on a Hewlett-Packard 1000F computer. On this small computer, the differential method generally requires at least 20 times as much computational time.

As a last remark, one can notice that, in TE polarization, the diffracted field $u^d = u - u^i$ is generated by currents located in the $y = 0$ plane and parallel to Oz. Consequently $u^d(x, -y) = u^d(x, y)$, which imposes that the Rayleigh coefficients R_n and T_n are such that $T_n = R_n + \delta_{n,0}$. This property can be and has been used to check the computer code.

5. PROBLEM OF A THIN GRATING LYING ON A SUBSTRATE

In this section, we generalize in two steps the theoretical results given in Section 2, supposing now that the grating G_{h_0} depicted on Fig. 1 is surrounded by two different homogeneous media with optical index ν^+ (for $y > 0$) and ν^- (for $y < 0$).

First Generalization

If the grating material is still an ohmic metal (described by ϵ_0, μ_0, and σ), we can consider the same limiting process as in Section 2. From theoretical considerations, which are the subject of a recent paper by our colleague Bouchitté[13] (and which have been verified with the help of the differential method), it turns out that, in the TE case, the transmission conditions [Eq. (2)] imposed to the limit field $u_0(x, y)$ are still valid. On the other hand, in the TM case and if $\nu(y)$ denotes the optical index [$\nu(y) = \nu^+$ if $y > 0$ and ν^- if $y < 0$], Eq. (2′) must be replaced by

$$\left[\!\left[\frac{1}{\nu^2}\frac{\partial u_0}{\partial y} \right]\!\right] = 0, \qquad [\![u_0]\!] = \frac{is}{k_0}\left(\frac{1}{\nu^2}\frac{\partial u_0}{\partial y}\right) f, \tag{10}$$

where the notation $[\![F]\!]$ stands for the jump at $y = 0$ of a function $F(x, y)$. Notice that this jump $[\![F]\!] = F(x, 0^+) - F(x, 0^-)$ is a function of x.

Second and Chief Generalization

Currently we have to deal often with materials that are lossy without being ohmic metals. They are in fact composite materials (such as rubber–carbon mixtures), macroscopically described by a relative permittivity whose real part is not unity. In this case we have of course to reconsider the limiting process described in Section 2. We denote now by $\epsilon = \epsilon' + i\epsilon''$ the complex permittivity (the square of the complex optical index) of grating G_{h_0} whose thickness is h_0. We impose that, if the thickness tends to zero, the permittivity varies in such a way that the product of the thickness and the permittivity remains equal to ϵh_0. The grating G_h has therefore a thickness h and a complex permittivity $\epsilon h_0/h$. When

$$l = h_0 \epsilon k_0, \tag{11}$$

theoretical considerations[13] show, and numerical experiments have confirmed, that the transmission conditions imposed on $u_0(x, y)$ by G_0 are now

$$[\![u_0]\!] = 0, \qquad \left[\!\left[\frac{\partial u_0}{\partial y} \right]\!\right] = -k_0 l f u_0 \qquad \text{in the TE case,} \tag{12}$$

$$\left[\!\!\left[\frac{1}{\nu^2}\frac{\partial u_0}{\partial y}\right]\!\!\right] = 0, \qquad [\![u_0]\!] = \frac{l}{k_0}\left(\frac{1}{\nu^2}\frac{\partial u_0}{\partial y}\right)_f$$

in the TM case. (13)

6. NUMERICAL STUDY OF A STACK OF LAYERS AND GRATINGS

In what follows, the periodic structure shown in Fig. 8 is called a grating layer. As already explained for a single grating, and provided its thickness h_0 is small enough, such a structure can be replaced by an infinitely thin grating that we call a grating interface. From numerical experiments quite similar to those described in Section 2, and illustrated by Fig. 2, it appeared to us that the domain of validity of this approximation is practically independent of the optical index of the dielectric that fills the dotted domain in Fig. 8. We have been concerned recently with the study of a stack of homogeneous layers intermixed with thin grating layers (Fig. 9a), all the grating layers having the same period d and a thickness smaller than some hundredths of a wavelength. We solved the problem as follows: We replaced each grating layer by the associated grating interface, and we dealt in fact with a stack of homogeneous layers separated either by regular plane interfaces or by grating interfaces (Fig. 9b). If such a structure is illuminated by a plane wave [Eq. (3)] the total field $u(x, y)$ is pseudoperiodic[12] and consequently can be expanded, with respect to x, in generalized Fourier series whose coefficients depend on y:

$$u(x, y) = \sum_n u_n(y)\exp(i\alpha_n x),$$

where α_n is still given by Eq. (6). In each layer, $u_n(y)$ is the sum of two terms, namely,

$$u_n^-(y) = A_n^- \exp(-i\beta_n y) \text{ and } u_n^+(y) = A_n^+ \exp(+i\beta_n y). \tag{14}$$

Of course, A_n^-, A_n^+, and β_n vary from one layer to another one. After truncation of the series, it is convenient to describe the field by a vector $\mathbf{U}$ (a column matrix) with $2P$ components, namely, P functions u_n^- and P functions u_n^+. Clearly $\mathbf{U}$ depends on y. Given two ordinates y_1 and y_2 ($y_1 > y_2$), a straightforward generalization of the well-known matrix theory of stratified media permits us to get a matrix $\mathbb{M}$ such that

$$\mathbf{U}(y_1) = \mathbb{M}\mathbf{U}(y_2).$$

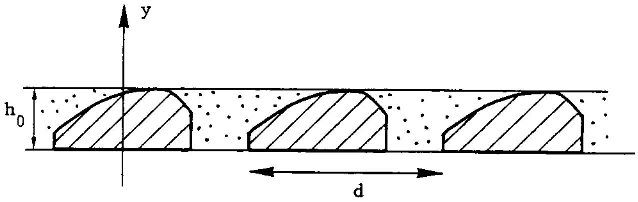

Fig. 8. Grating-layer for which the hatched and the dotted areas are filled with absorbing materials (relative permittivity $\epsilon = \epsilon' + i\epsilon''$) and dielectric materials, respectively.

a

b

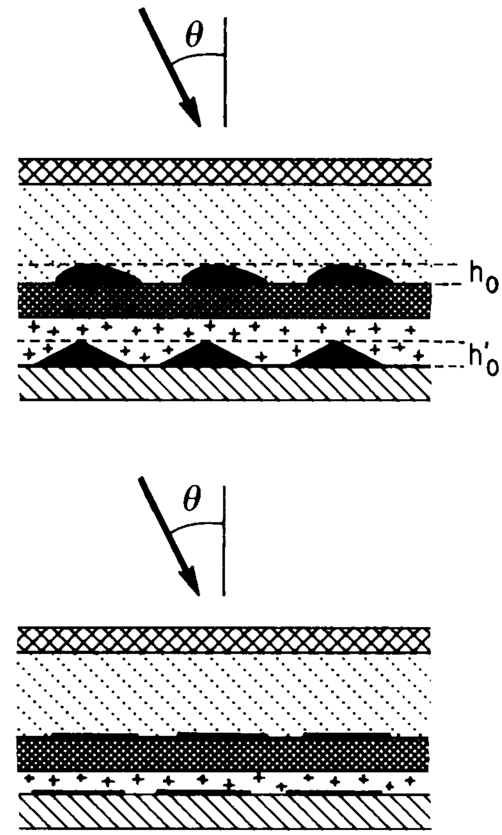

Fig. 9. a, Stack of homogeneous layers intermixed with thin grating layers. b, Equivalent structure used for computation; each grating layer has been replaced by the associated grating interface.

Such a matrix is obtained as a product of three kinds of matrices, namely,

$\mathbb{L}$ matrices associated with the propagation inside a layer,
$\mathbb{I}$ matrices associated with the crossing of grating interfaces,
$\mathbb{I}'$ matrices associated with the crossing of regular plane interfaces.

$\mathbb{L}$ matrices are diagonal matrices whose determination is obvious. Therefore the only problem is to write the coefficients of $\mathbb{I}$ matrices since a regular plane interface is a particular case of a grating interface. We give in Appendix A the transfer relations concerning u_n^+ and u_n^- if a grating interface is crossed. These relations result from the transmission conditions [Eqs. (12) and (13)]. If the reader finds it necessary, they can be put in matrix form. Anyway, it is understood that, for any stack of layers and thin gratings located between the ordinates $y_{\min}$ and $y_{\max}$, one can construct a matrix $\mathbb{M}$ such that

$$\mathbf{U}(y_{\max}) = \mathbb{M}\mathbf{U}(y_{\min}). \tag{15}$$

It is convenient to split $\mathbf{U}$ into two blocks U^- and U^+, respectively associated with downward and upward waves. Then Eq. (15) can be written as

$$\begin{bmatrix} U^-(y_{\max}) \\ U^+(y_{\max}) \end{bmatrix} = \begin{bmatrix} \mathbb{M}_{11} & \mathbb{M}_{12} \\ \mathbb{M}_{21} & \mathbb{M}_{22} \end{bmatrix} \begin{bmatrix} U^-(y_{\min}) \\ U^+(y_{\min}) \end{bmatrix}. \tag{15'}$$

Solving a problem consists of finding the columns $U^+(y_{max})$ and $U^-(y_{min})$ if $U^-(y_{max})$ and $U^+(y_{min})$ are known. This is a classical problem of linear algebra that reduces to matrices products and matrix inversions. If the stack is illuminated from the top by a plane wave, $U^-(y_{max})$ reduces to a number and $U^+(y_{min})$ vanishes; if the stack lies on a perfectly conducting substrate, we have in TE polarization $U^+(y_{min}) = -U^-(y_{min})$, etc. We will not enter the details, and obviously several numerical treatments are possible. Anyway, owing to short computation times, this approximate method is attractive and can be implemented on a microcomputer. Unfortunately, and as far as we know, the upper bound of the error is not known. In the course of a given study, we must, from time to time, compare our data with those provided by an exact method.

As an illustration, we will give some data for two particular structures; possibly they can be used to compare our results with other ones. At first, consider the structure depicted on Fig. 10, which contains two grating layers having the same thickness $h_0 = 0.01\lambda_0$; the permittivity of the hatched region is $\epsilon = 8 + i3.5$, a value from a recent paper[14] devoted to microwave absorbers at 9.5 GHz. The optical index ν has been arbitrarily chosen as 1.5, and $t = \lambda_0/10$. For both gratings the pitch is $d = 4/3\lambda_0$, and the widths of the strips are $c = 0.75d$ and $c = 0.5d$ for the triangular grating and the rectangular grating, respectively. The efficiency curves are given in Fig. 11 for TE polarization. We recall our definition of the reflected efficiency in order n: $e_n = |R_n|^2\beta_n/\beta_0$ if we use the notations of Eq. (4). Figure 12 represents an array of thin strips with rectangular cross section and thickness h_0, separated from an infinitely conducting substrate by a dielectric. The strips are made with a material whose relative permittivity is $\epsilon = \epsilon' + i\epsilon''$ and correspond, therefore, to a parameter $l = k_0h(\epsilon' + i\epsilon'')$. We suppose that $k_0h\epsilon' \ll 1$ and $k_0h\epsilon'' = 1$. Under these conditions and taking $l = i$, we obtain, for different values of c/d, the curves of Fig. 13. If $c/d = 1$, the structure reduces to the so-called Salisbury screen. The condition $k_0h\epsilon'' = 1$ corresponds to a square resistance of $120\pi = 377\ \Omega$/square, and one can verify that, for $c/d = 1$,

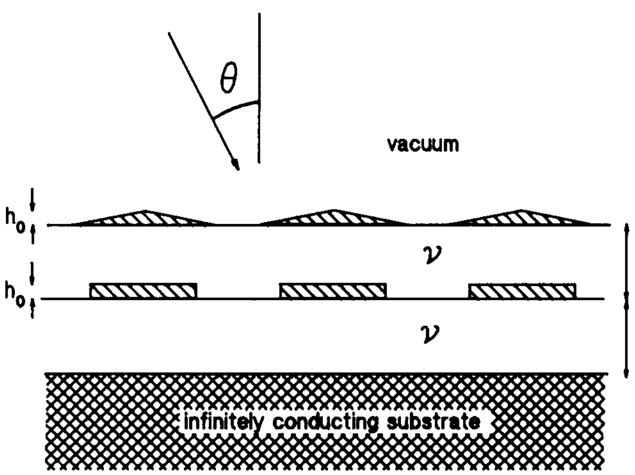

Fig. 10. Two grating layers separated by dielectric layers.

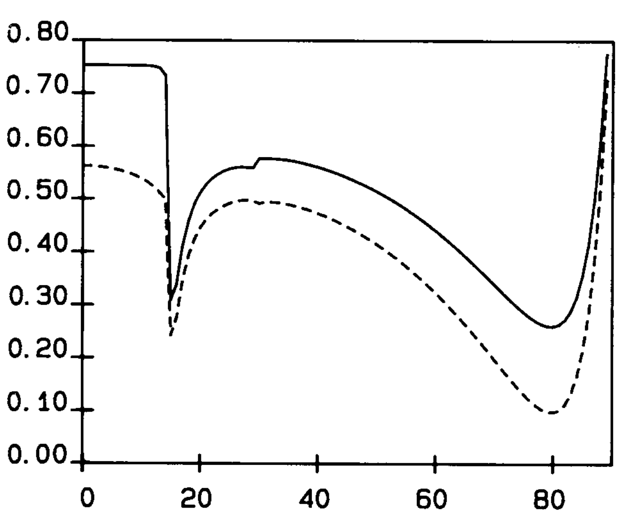

Fig. 11. Efficiency curves: dashed curve, reflected efficiency in the zero order versus the angle of incidence θ; solid curve, sum of the efficiencies in the reflected orders versus θ.

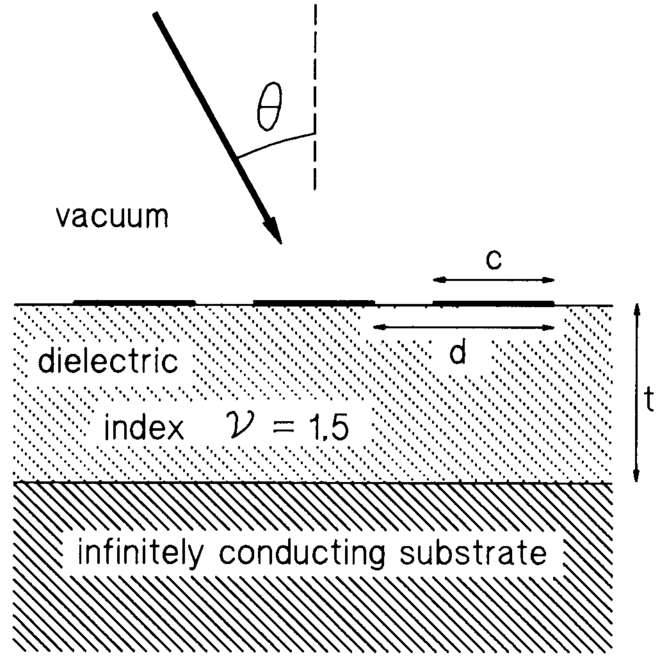

Fig. 12. Modified Salisbury screen: $t = \lambda_0/4\nu$, $\lambda_0 = 0.75d$.

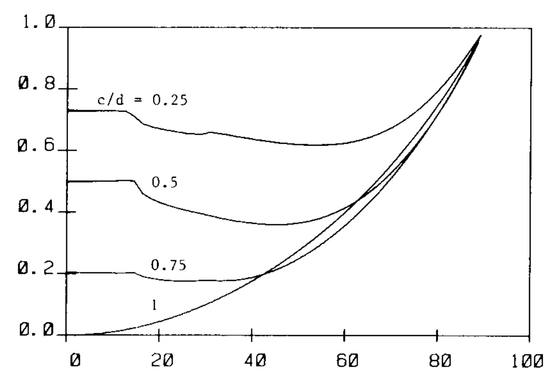

Fig. 13. Modulus of the reflection coefficient in the zero order (i.e., $|R_0|$ using notations of Section 4) versus θ.

our curve is in agreement with that given in radar literature (Ref. 15, p. 244).

7. CONCLUSION

A numerical method has been proposed for the study of diffraction and absorption by sufficiently thin gratings. The strip cross section, to a large extent, is arbitrary, and the grating material does not need to verify Ohm's law, as do the usual metals in the microwave range. It can be, for example, a rubber–carbon mixture. The case of arrays intermixed with dielectric layers (Fig. 9a) is also solved by using a generalized scattering matrix theory. The numerical implementation is easy, and the resulting code can be used to investigate the properties of rather complicated periodic structures (Fig. 9a) if the complex permittivities of the different materials are known. Owing to the short computation time, this method is especially recommended to analyze the influence of a particular parameter on the behavior of the structure, as a whole. If necessary, the validity of the approximation can be checked, for some points, by comparison with a more rigorous but also more time-consuming integral or differential method.

APPENDIX A

Let us consider a grating layer (Fig. 8) surrounded by two layers (optical index ν^+ for the upper layer and ν^- for the lower one). Function $f(x) = \sum_n f_n \exp(in2\pi x/d)$ describes the shape of the cross section. If this grating layer is sufficiently thin, it can be replaced by a grating interface placed at a certain ordinate y_0 and characterized by the complex number $l = k_0 h_0 \epsilon$. In each layer, the total field can be written as $u(x, y) = \sum_n [u_n^-(y) + u_n^+(y)]\exp(i\alpha_n x)$. We put $(\beta_n^+)^2 = k_0^2(\nu^+)^2 - \alpha_n^2$ and $(\beta_n^-)^2 = k_0^2(\nu^-)^2 - \alpha_n^2$ with $\beta_n^\pm > 0$ or $\beta_n^\pm/i > 0$. Recalling that in each layer u_n^- and u_n^+ are given by Eqs. (14), the boundary conditions [Eqs. (12) or (13)] yield

$$u_n^-(y_0^+) = s_n u_n^-(y_0^-) + d_n u_n^+(y_0^-) + \frac{k_0 l}{2i\beta_n^+}\sum_m f_{n-m}[u_m^-(y_0^-) + u_m^+(y_0^-)],$$

$$u_n^+(y_0^+) = d_n u_n^-(y_0^-) + s_n u_n^+(y_0^-) - \frac{k_0 l}{2i\beta_n^+}\sum_m f_{n-m}[u_m^-(y_0^-) + u_m^+(y_0^-)]$$

in TE polarization and

$$u_n^-(y_0^+) = s_n u_n^-(y_0^-) + d_n u_n^+(y_0^-) - \frac{l}{2ik_0(\nu^-)^2}\sum_m f_{n-m}\beta_m^-[-u_m^-(y_0^-) + u_m^+(y_0^-)],$$

$$u_n^+(y_0^+) = d_n u_n^-(y_0^-) + s_n u_n^+(y_0^-) - \frac{l}{2ik_0(\nu^-)^2}\sum_m f_{n-m}\beta_m^-[-u_m^-(y_0^-) + u_m^+(y_0^-)]$$

in TM polarization, where

$$s_n = \frac{\beta_n^+ + \beta_n^-}{2\beta_n^+}, \qquad d_n = \frac{\beta_n^+ - \beta_n^-}{2\beta_n^+}$$

in TE polarization or

$$s_n = \frac{\beta_n^+/(\nu^+)^2 + \beta_n^-/(\nu^-)^2}{2\beta_n^+/(\nu^+)^2}, \qquad d_n = \frac{\beta_n^+/(\nu^+)^2 - \beta_n^-/(\nu^-)^2}{2\beta_n^+/(\nu^+)^2}$$

in TM polarization.

As usual, $u_n^-(y_0^+)$ stands for the limit of $u_n^-(y)$ if y tends to y_0 with $y > y_0$, and the coefficients s_n, appearing here are of course not connected with the parameter s defined by Eq. (1). It must be noticed that the index of the dotted domain (Fig. 8) does not appear in the boundary conditions.

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