Green's function for biperiodic gratings. Application to the study of directive antennas using metallic photonic crystals properties.

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Summary : Three-dimensional electromagnetic problems require huge computing resources. One way to go through is to consider periodic structures in order to reduce the investigation domain to one cell of the structure. The Green's function which appears in this case involves expansions which converge very slowly. Combining several representations of the Green's function and using Shanks' transform, we have developed an efficient algorithm. Using an integral method based on Harrington's formalism, we model a structure made of a non-periodic source covered or surrounded by metallic biperiodic grids. With in aim the design of compact directive antennas in Ku band, we propose two structures. The first one is a Pérot-Fabry like cavity, and the second one uses the properties of photonic crystals near the band edge.

Abstract : L'étude électromagnétique de problèmes tridimensionnels nécessite des ressources informatiques énormes. Une manière de contourner cette difficulté est de considérer le cas de structures périodiques pour ramener le domaine d'étude à une période. La fonction de Green bipériodique qui intervient alors fait intervenir des séries de convergence très lente. En combinant plusieurs représentations de la fonction de Green et en utilisant la transformation de Shanks, nous avons mis au point un algorithme de calcul très efficace. A partir du formalisme de Harrington, nous modélisons une structure constituée d'une source surmontée ou entourée d'un ensemble de grilles métalliques. Deux types de structures permettent alors de modéliser des antennes directives compactes en bande Ku : l'une utilise une cavité résonnante de type Pérot-Fabry, l'autre est basée sur les propriétés des cristaux photoniques en bord de bande interdite.

I. INTRODUCTION.

Three-dimensional electromagnetic problems require huge computing resources. One way to go through is to consider periodic structures in order to reduce the investigation domain to one cell of the structure. Many numerical methods, such as integral methods, require the computation of a Green's function. Unfortunately, the more straightforward expressions of periodic Green's functions lead to very slow converging series. Numerous works have been devoted to the Green's function for one-dimensional gratings. The acceleration of the convergence can result from different transformations: Kummer's transform combined with Poisson's transform, numerical non linear transforms such as Shanks [1], Levin or ρ-transform. An alternative solution is to use the so-called Lattice Sums [2]. In this paper, we focus on the efficient computation of the Green's function for doubly periodic arrays (used for instance in crossed grating problems). Fewer works concern this problem. We combine the techniques reported in the earlier studies by Jorgenson et al [3] and by Singh et al [4]. By this way, we obtain different methods to compute this Green's function. None of these methods is optimum for all positions of the observation point. A systematic numerical study allows us to define the regions of space where each method offers the best performances. The resulting Fortran subroutine takes these considerations into account in order to choose automatically the most efficient method [5].

With in aim the design of compact directive antennas in Ku band, we propose two structures made of metallic biperiodic grids which form a photonic crystal. The first one is a Pérot-Fabry like cavity, and the second one uses the properties of photonic crystals near the band edge. In this case, we use an adaptation of the integral method based on Harrington's formalism [6], taking advantage of our Green's function.

II. GREEN'S FUNCTION.

1. Spatial and spectral forms

The doubly periodic Green's function described here is the solution verifying an outgoing wave condition of the Helmholtz equation with doubly pseudo-periodic elementary sources, and writes in the following "spatial" form:

$$G(x,y,z) = -\frac{1}{4\pi}\sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty}\frac{e^{ikr_{nm}}}{r_{nm}}e^{i\alpha_0 nd_x + i\beta_0 md_y} \quad (1)$$

where r_{nm} is the distance from the "source" located at point $(nd_x, md_y, 0)$ to the observation point (x,y,z), d_x and d_y are the periods along x and y axes, α_0 and β_0 are the pseudo-periodicity coefficients, $k = 2\pi/\lambda$ where λ represents the wavelength. From calculus using Fourier series, we get from (1) another expansion for G, that we denote by "spectral" form:

$$G(x,y,z) = \frac{1}{2i\,d_x d_y}\sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty}\frac{e^{i\gamma_{nm}|z|}}{\gamma_{nm}}e^{i\alpha_n x + i\beta_m y} \quad (2)$$

where

$$\alpha_n = \alpha_0 + n\frac{2\pi}{d_x}, \qquad \beta_m = \beta_0 + m\frac{2\pi}{d_y},$$

$$\gamma_{nm}^2 = k^2 - \alpha_n^2 - \beta_m^2$$

and choosing γ_{nm} or γ_{nm}/i as a positive number.

2. First mixed form

Let us denote S_{1nm} the term of the series (1). We obtain another series by subtracting and adding a term $\tilde{S}_{1nm}$ where r_{nm} is replaced by a slightly different value:

$$\tilde{r}_{nm} = \sqrt{(x - n\,d_x)^2 + (y - m\,d_y)^2 + \left(|z| + u\sqrt{d_x d_y}\right)^2}$$

, where u is a positive parameter to be optimized. The series whose term is $S_{1nm} - \tilde{S}_{1nm}$ converges faster than the initial one. The series whose term is $\tilde{S}_{1nm}$ is transformed in a "spectral" form similar to (2):

$$\frac{1}{2i\,d_x d_y} \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \frac{e^{i\,\gamma_{nm}\left(|z| + u\sqrt{d_x d_y}\right)}}{\gamma_{nm}} e^{i\alpha_n x + i\beta_m y} \qquad (3)$$

Note that the parameter u gives a z-translation which enables us to get a fast convergence.

3. Second mixed form

A similar technique is used to obtain a fourth expression of G, starting from (2). Let us denote S_{2nm} the term of the series (2). We obtain another series by subtracting and adding a term $\tilde{S}_{2nm}$ where γ_{nm} is replaced by a slightly different value $\tilde{\gamma}_{nm}^2 = -v^2 - \alpha_n^2 - \beta_m^2$, where v is a real parameter to be optimized. The series whose term is $S_{2nm} - \tilde{S}_{2nm}$ converges faster than the initial one. The series whose term is $\tilde{S}_{2nm}$ is transformed in a "spatial" form similar to (1):

$$-\frac{1}{4\pi} \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \frac{e^{-v\,r_{nm}}}{r_{nm}} e^{i\alpha_0 n d_x + i\beta_0 m d_y} \qquad (4)$$

The convergence is due to $\exp(-v\,r_{nm})/r_{nm}$.

4. Numerical technique and results

On the two mixed forms, we use the Shanks' transform [1] in order to accelerate numerically the convergence. It appears that none of previous expansions is able to give fast and accurate results for all values of the opto-geometrical parameters (the most sensitive ones being the position of the observation point, the wavelength and the periods d_x and d_y). Fortunately, it is always possible to find at least one expansion giving satisfactory results in each particular case. From numerous numerical investigations, we have been able to find empirical rules for the determination of the best expansion to use.

In figure 1a, we plot the percentage of points that do not meet the accuracy criterion, i.e. the obtained accuracy is less than the requested one ε. The parameters are $d_x = d_y = \lambda = 1$, $\alpha_0 = k\sin(\pi/4)$, $\beta_0 = 0$. The Green's function is computed over 10,000 points equally spaced in x and y inside the first period, and logarithmically spaced from $z = 10^{-6}$ to $z = 1$. The second mixed form with Shanks' transform gives good results as long as $\varepsilon > 10^{-6}$, but the number of false results increases as ε decreases. Our numerical code gives some false results especially for very small values of ε. The most robust method is given by the first mixed form with Shanks' transform, but it is not the faster, as it can be seen on figure 1b. From this last point of view, our code offers the best performances.

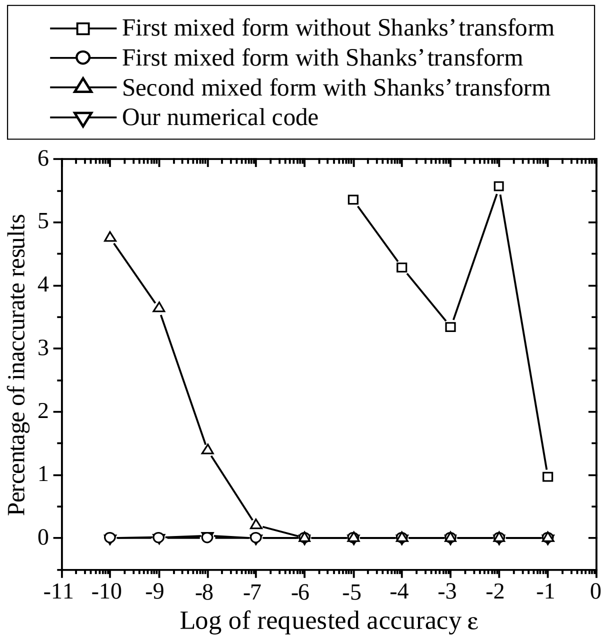

Fig 1a: percentage of inaccurate results for the various cases

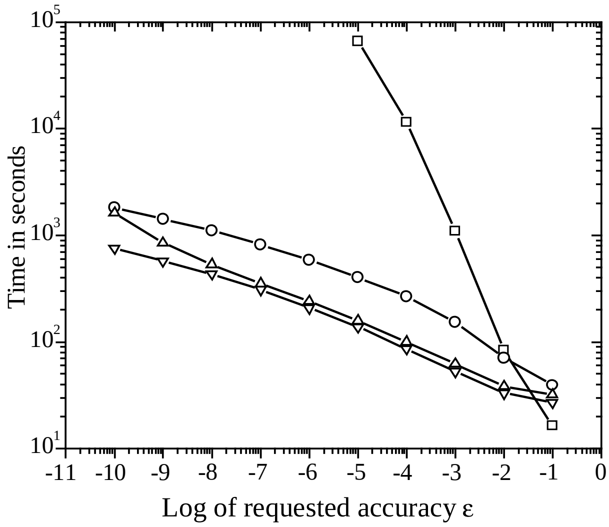

Fig 1b: Computation time for the various cases

The reader interested in more details can refer to our paper [5].

III. ANTENNA MODELING.

Using an integral method based on Harrington's formalism, we model a structure made of a non-periodic source (patch, monopole,…) laying on a ground plane and covered or surrounded by metallic biperiodic grids. These grids represent a photonic crystal and are made of infinitely conducting wires whose diameter is small compared to the wavelength λ. The field emitted by the source in free space is represented by a plane wave packet using FFT. Each of these plane waves gives rise to a pseudo-periodic problem, i.e. a classical grating problem.

The first structure is based on a Pérot-Fabry like planar resonant cavity. A ground plane acts as one of the mirror of the cavity, whereas the opposite mirror is made of two metallic grids. The parameters of these grids are optimized in order to obtain a suitable directivity for the emitted field: $d_x = d_y = 5.8\,\text{mm}$, radius of the wires 0.26mm, spacing between the ground plane and the first grid: 9.34mm, spacing between the first and the second grids: 5.8mm, for λ = 21.4mm. This structure is excited by a patch located in the cavity. Fig. 2 gives a sketch of patch, showing the feeding point, and the principal direction of the surface currents (arrows). θ and φ are the usual spherical coordinates angles. The radiation patterns will be drawn in the two planes $x = 0$ and $y = 0$. Note that due to the symmetry of the patch, the radiation pattern will be symmetrical in the plane $y = 0$, but not in the plane $x = 0$.

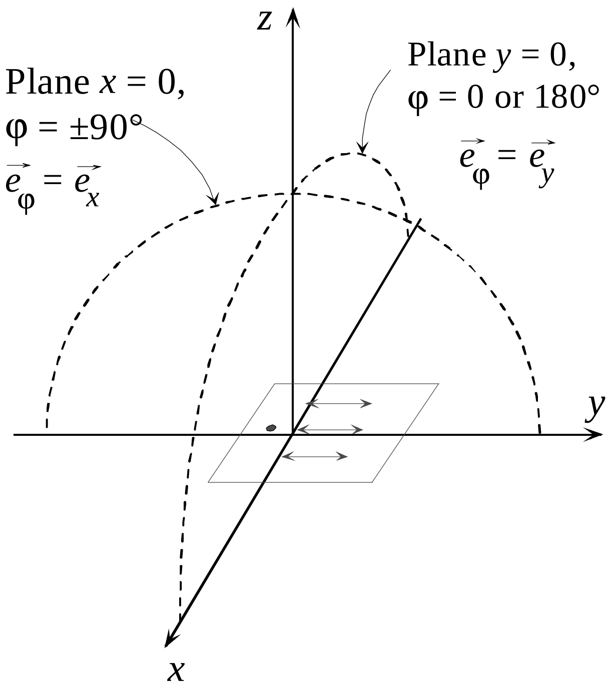

Fig 2: Notations.

Fig. 3 shows the radiation patterns of the device in dB scale. It shows that the emission is concentrated in a narrow lobe. The arrays of wires do not affect the polarization of the emitted field, which stays linearly polarized in the lobe. The half-power beamwidths are 2×5.1° in the plane $y = 0$ and 2×5.5° in the plane $x = 0$. Note that all the radiation patterns are in arbitrary units.

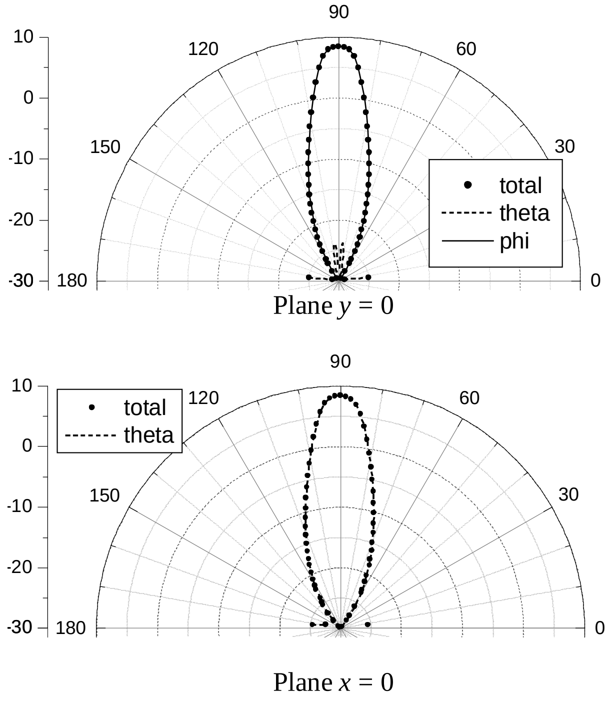

Fig 3: Radiation patterns of the Pérot-Fabry like structure.

The second solution uses the specific properties of a metallic photonic crystal at the band edge. Metallic photonic crystals present two kinds of bandgaps: a large low-frequency gap, and smaller gaps for higher frequencies. In this study, we make use of the filtering properties of the large low-frequency gap. A pretty way for the investigation of these filtering properties is based on the dispersion diagrams of Bloch modes [7]. By this way, we have determined convenient parameters for the structure. The resulting structure is made of a ground plane covered by a photonic crystal made of 6 metallic crossed grids. The radiative element is an embedded patch.

In this case, the parameters are $d_x = d_y = 5.8\,\text{mm}$, radius of the wires 0.277mm, spacing between the ground plane and the first grid: 3.4mm, spacing between each of the grids: 6.8mm, and λ = 21.1mm. Figures 4 and 5 show the radiation patterns of the device in dB scale. Here again, the emission is concentrated in a narrow lobe: the half-power beamwidths are 2×2.0° in the plane $y = 0$ and 2×2.6° in the plane $x = 0$.

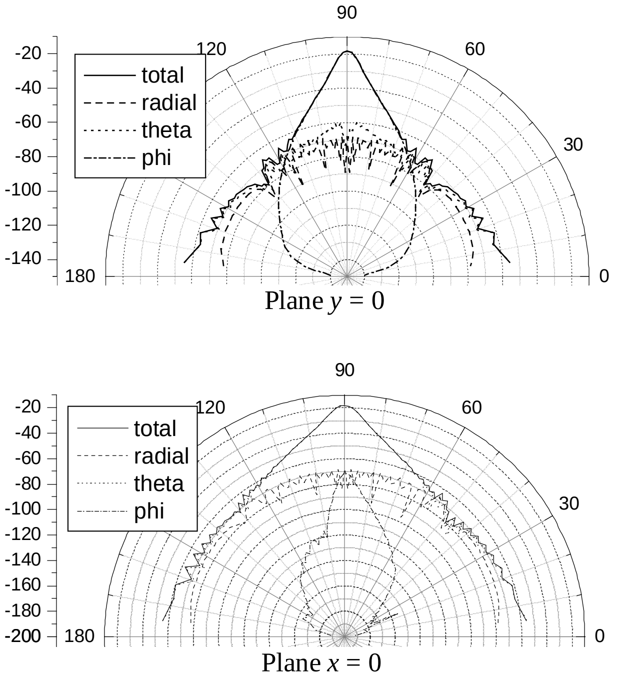

Fig 4: Radiation patterns of the photonic crystal structure.

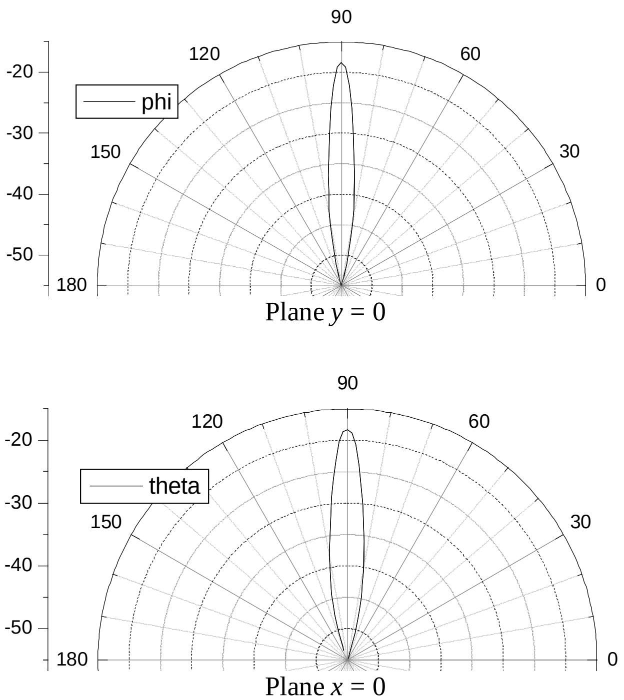

Fig 5: Same as Fig.4, enlarged scale.

CONCLUSION

A complete modeling of antenna devices requires the computation of the impedance. The method used in this paper is not able to give this information. This is the reason why we are presently developing another way to circumvent the numerical problems linked with the large number of unknowns in the 3D structure. The idea is to take advantage of the partial block-Toeplitz structure of the impedance matrix involved in the Harrington's formalism, combined with the use of an iterative solver. Compared with the FFT decomposition described in section III, this technique will allow us first to take into account the interaction between the source and the arrays of wires, and second to model a finite structure.

An experimental study is also in progress, and we expect that we will be able to confirm in the next months the predictions of this work.

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