

# DIRECTIVE EMISSION FROM PHOTONIC QUASICRYSTALS WITH 12-FOLD SYMMETRY

*A. Micco, V. Galdi, V. Pierro*

*Waves Group, Department of Engineering, University of Sannio, I-82100 Benevento, Italy.*

*[alemicco@yahoo.it](mailto:alemicco@yahoo.it), [vgaldi@unisannio.it](mailto:vgaldi@unisannio.it), [pierro@unisannio.it](mailto:pierro@unisannio.it)*

*A. Della Villa, F. Capolino*

*Department of Information Engineering, University of Siena, I-53100 Siena, Italy.*

*[dellavilla@gmail.com](mailto:dellavilla@gmail.com), [capolino@dii.unisi.it](mailto:capolino@dii.unisi.it)*

*S. Enoch, G. Tayeb*

*Institut Fresnel, CNRS 6133, Université Paul Cézanne Aix-Marseille III,*

*13397 Marseille cedex 20, France. [stefan.enoch@fresnel.fr](mailto:stefan.enoch@fresnel.fr), [gerard.tayeb@fresnel.fr](mailto:gerard.tayeb@fresnel.fr)*

**Abstract:** This paper deals with the study of the electromagnetic radiation from electric line sources embedded in two-dimensional photonic “quasicrystal” slabs made of dielectric rods arranged according to a 12-fold symmetric aperiodic tiling. Preliminary numerical results, based on a rigorous full-wave method, seem to indicate significant improvements, in terms of sidelobe levels, as compared to previous studies (restricted to 5-fold-symmetric Penrose-type geometries).

## INTRODUCTION AND BACKGROUND

During the last two decades, there has been a growing interest in the study of the physical properties and applications of *aperiodically-ordered* structures, related to the concept of “quasicrystals” in solid-state physics, in many fields of science and technology (see [1] for a recent review on the subject, and [2] for an up-to-date bibliography database). In electromagnetic (EM) engineering, use of *photonic quasicrystals* (PQCs) has been proposed in several applications, involving lasers, negative refraction and superlensing, high-impedance substrates etc. (see [3-6], and the references therein, for a sparse sampling).

In [7], we presented a comparative study of two-dimensional (2-D) QPCs made of dielectric rods arranged according to selected *aperiodic tiling* geometries, featuring different types and degrees of order and symmetry. In particular, we studied in detail the radiation from a line source embedded in a PQC slab with Penrose-type (5-fold symmetry) lattice. Results showed the possibility of achieving *moderate* directivity at three frequencies located nearby the (main and secondary) bandgaps, with the lowest frequency smaller (~68%) than the (single) value observed for a periodic photonic crystal slab with comparable size and filling factor. However, the sidelobe level (~-5dB) was still rather high.

In this paper, we show that significant improvements can be obtained by using PQCs with 12-fold symmetry, similar to those considered in [6] for negative refraction and superlensing applications.

## DIRECTIVE EMISSION FROM PQCs WITH 12-FOLD SYMMETRY

*Geometry and Parameters.* As in [7], we consider a 2-D ( $z$ -invariant) geometry (see Fig. 1) involving a slab of 435 dielectric rods located at the vertexes of a 12-fold-symmetric aperiodic tiling (see the inset in Fig. 1) generated combining square and equilateral triangles according to the Stampfli rules [6,8]. The total size of the slab is  $107a \times 4.3a$ , with  $a$  being the period of an equivalent periodic crystal slab having the same total size and filling factor. The rods have radius  $r = 0.15a$  and relative permittivity  $\epsilon_r = 12$ . A time-harmonic electric line source is located at

( $x = 0, y = -0.52a$ ), with a slight vertical displacement from the symmetry center of the tiling (at  $x = y = 0$ ), so as to avoid the placement inside a dielectric rod.

*Preliminary Results.* Figure 2 shows the EM response of the PQC of interest computed via a well-established full-wave numerical technique based on a multipolar Fourier-Bessel expansion [9]. Specifically, Fig. 2(a) shows the normalized *radiativity* (cf. [7, Eq. (2)]) from a 257-rod square-shaped PQC slab as a function of the normalized frequency  $a/\lambda_0$  (with  $\lambda_0$  denoting the free-space wavelength), from which at least three bandgap regions can readily be identified as pronounced dips. Figure 2(b) shows the normalized radiation patterns in the top halfspace, for a line source embedded in the 435-rod PQC slab shown in Fig. 1, at three representative frequencies ( $a/\lambda_0 = 0.220, 0.292, 0.678$ ) for which some moderate-to-high directive behavior is achieved. It is observed that the two lowest operational frequencies turn out to be smaller than the lowest one in [7]. Moreover, while the sidelobe level at the lowest frequency ( $a/\lambda_0 = 0.220$ ) is comparable with those in [7], it is *considerably lower* ( $\sim -10$ dB) at the other two frequencies ( $a/\lambda_0 = 0.292, 0.678$ ).

## CONCLUSIONS AND PERSPECTIVES

We presented a study of directive emission from line source embedded in PQC slabs with 12-fold symmetry. Preliminary results turn out to be significantly better, in terms of sidelobe level, than those presented in [7] for the Penrose (5-fold symmetry) case. It should be noticed that the PQC slab in [7] was randomly extracted from the Penrose tiling, whereas the PQC slab here was extracted from the 12-fold-symmetric tiling preserving the (local) center of symmetry. Moreover, we cannot rule out the possibility that the improvement stems from a better matched choice of the slab size and/or filling factor. Accordingly, current and future investigations are aimed at better understanding the role of *local* vs. *global* symmetry, as well as the slab geometrical parameters, in the directive emission properties. Preliminary results, for 12-fold-symmetric PQCs, seem to indicate a general degradation of the directivity when the line source is horizontally displaced from the  $x=0$  symmetry axis. On the other hand, vertical displacements along the  $y$ -axis seem to yield, in certain cases, *lower* sidelobe levels. Similar parametric studies are being carried out for other (Penrose, 8-fold) PQC geometries. Also of interest is the development of leaky-wave-based analytic parameterizations of the radiated field.

## REFERENCES

- [1] Maciá, E., "The role of aperiodic order in science and technology," *Rep. Progr. Phys.*, **vol. 69**, no. 22, Feb. 2006, pp. 397-441.
- [2] <http://www.quasi.iastate.edu/bib.html>, maintained by the Quasicrystal Research Group at Iowa State University, IA, USA.
- [3] Notomi, M., H. Suzuki, T. Tamamura, and K. Edagawa, "Lasing action due to the two-dimensional quasiperiodicity of photonic quasicrystals with a Penrose lattice," *Phys. Rev. Lett.*, **vol. 92**, no. 12, Mar. 2004, 123906.
- [4] Li, H. Q., Z. H. Hang, Y. Q. Qin, Z. Y. Wei, L. Zhou, Y. W. Zhang, H. Chen H, and C. T. Chan, "Quasi-periodic planar metamaterial substrates," *Appl. Phys. Lett.*, **vol. 86**, no. 12, Mar. 2005, 121108.
- [5] Wang, Y. Q., S. S. Jian, S. Z. Han, S. Feng, Z. F. Feng, B. Y. Cheng, and D. Z. Zhang,

- “Photonic band-gap engineering of quasiperiodic photonic crystals,” *J. Appl. Phys.*, **vol. 97**, no. 10, May 2005, 106112.
- [6] Feng, Z., X. Zhang, Y. Wang, Z.-Y. Li, B. Cheng, and D.-Z. Zhang, “Negative refraction and imaging using 12-fold-symmetry quasicrystals,” *Phys. Rev. Lett.*, **vol. 94**, no. 24, June 2005, 247402.
- [7] Della Villa, A., V. Galdi, F. Capolino, V. Pierro, S. Enoch, and G. Tayeb, “A comparative study of representative categories of EBG dielectric quasicrystals,” *IEEE Antennas Wireless Propagat. Lett.*, **vol. 5**, 2006, pp. 331-334.
- [8] Oxborrow, M. and C. L. Henley, “Random square-triangle tilings: A model for twelvefold-symmetric quasicrystals,” *Phys. Rev. B*, **vol. 48**, no. 10, Sept. 1993, pp. 6966-6998.
- [9] Felbacq, D., G. Tayeb, and D. Maystre, “Scattering by a random set of parallel cylinders,” *J. Opt. Soc. Am. A*, **vol. 11**, no. 9, Sept. 1994, pp. 2526-2538.

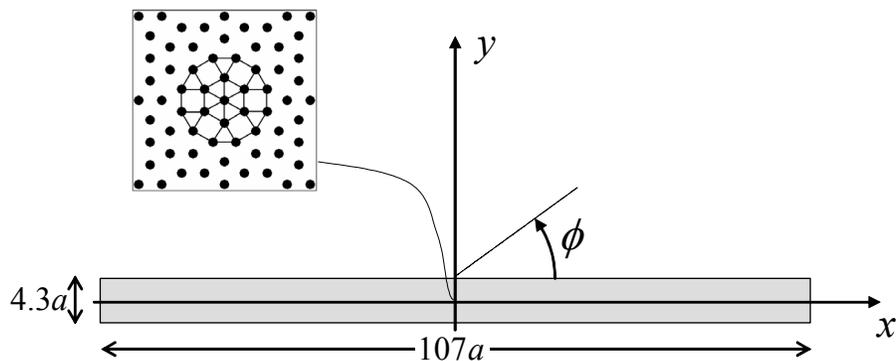


Figure 1 – Problem geometry (explained in the text).

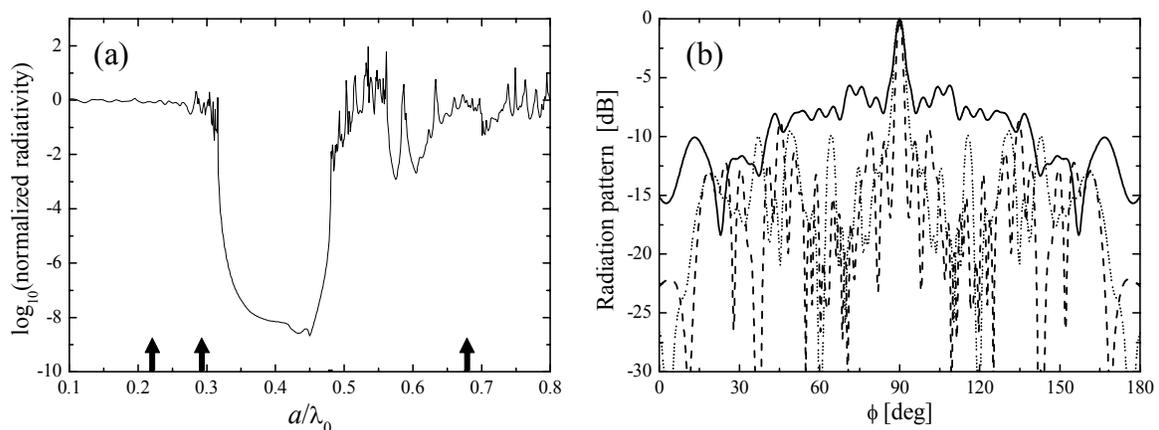


Figure 2 – Electromagnetic response. (a) Normalized radiativity, as a function of normalized frequency, for a 257-rod square-shaped PQC slab. (b) Radiation patterns from a line source (at  $x=0, y=-0.52a$ ) embedded in the 435-rod PQC slab shown in Fig. 1, at three representative frequencies (marked as arrows in (a)). Solid curve:  $a/\lambda_0=0.220$ ; dashed curve:  $a/\lambda_0=0.292$ ; dotted curve:  $a/\lambda_0=0.678$ .